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Essays on Environment, Natural Resource, Growth and Development

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Essays on environment, natural resource, growth and development

by

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Chapter 1

General Introduction

It is not from the benevolence of the butcher, the brewer or the baker, that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love, and never talk to them of our own necessities but of their advantages.

— *Adam Smith*

1.1 Introduction

The principle of economics begins with the "invisible hand", believing that free market can be efficient in allocating resources. But in practice market may fail. Market does not take into account externalities, like damaging effects of pollution. The various frictions in the real world, like asymmetric information, may also invalidate the efficiency of "invisible hand". It is beyond controversy that government intervention is necessary to remedy those market failures. But in many situations, how to design good policies to encounter market failure is not straightforward. Moreover, if without careful study of their consequences, policies intended to do good deed may bring negative results, leading to government failure. The three essays in this dissertation aim to examine government policies addressing on externality of pollution and imperfect credit market, where policy impact or policy design is not trivial as one would think at first blush.

As a response to global climate change and high energy prices, major economies throughout the world are promoting the development of renewable energies such as biofuels, wind and solar energies. But what is the economic as well as greenhouse gas (GHG) impacts of government supports for renewable energies? One may be easily lead to think that government support can help substitute clean renewable energies for fossil fuels and therefore benefit the environment. However that convention ignores an important fact that owners of fossil fuels would respond to renewable energy policies and that could generate unfavorable consequence. Sinn (2008) argues that policies reducing demand for fossil fuels, e.g. increasing tax rate on carbon, improving energy efficiency, and increasing the use renewable energies, could lead to overextraction of fossil fuels in near future and exacerbate the problem of climate change, and he calls it Green Paradox. In Chapter 2, by recognizing the capacity constraints of renewable energies, we generalize renewable energy policies and examine their climate change impacts. In this Chapter, we can learn that impacts of renewable energy policies on climate change is not trivial as one would expect and both capacity constraints of renewable energies and market power play important roles in it.

A particular phenomena we can observe in emerging countries, like China, is the coexistence of high growth, high pollution and high savings. Could there exist a mechanism to connect them? An empirical evidence from Chinese data reveals that higher pollution levels are associated with high savings rates. Since savings rate is critical for the long-run growth, could that mechanism be a mutually-reinforcing pollution-growth nexus? We address that question and conduct policy analysis in Chapter 3. On the other hand, pollution hurts the health of people. For instance, air pollution has long-term and short-term effects on morbidity and mortality associated with respiratory and cardiovascular illness. Moreover, according to contingent valuation studies of willingness to pay for pollution reductions, like World Bank (2007) and U.S. EPA (1997), health damage is the costliest and in term of monetary value it accounts more than 90% of all damages pollution generates. Therefore if we want to explore the connection between pollution and growth, the health effect is critical. Taking account of the health effect, Chapter 3 constructs an overlapping-generations model in which agents save more in response to the higher pollution-induced health risk and the increased saving in turn leads to more investment, and thus more pollution. Then based on that benchmark model, Chapter 3 derives important policy implication.

Human capital is important for an economy to grow. But in most countries, students, especially those from poor families, are difficult to borrow from the credit market and thus are generally short of funds for educational investments. That mainly is because future labor income cannot be collateralized due to the inalienability of human capital. Given that imperfect credit markets, can public policy restore human capital investment to socially optimal levels? At first sight, it may appear that a carefully-chosen public subsidy to education could ensure optimal accumulation of human capital. But it has been shown that education subsidy alone is generally not enough to replicate complete market solutions. Following Boldrin and Montes (2005) and Andolfatto and Gervais (2006), Chapter 4 generalizes the answers to the above question in the framework of endogenous borrowing constraints.

1.2 Dissertation organization

The rest of the dissertation is organized as follows. Chapter 2, 3 and 4 are the three independent papers as discussed. Chapter 5 concludes the dissertation. Appendix A gives additional figures and definition used in Chapter 2. Appendix B explains the data used in Chapter 3 and presents their descriptive statistics. Proofs of major results in Chapter 2, 3 and 4 are provided in Appendix C.

Chapter 2

Climate Change Impacts of Renewable Energy Policies: the Roles of Capacity Constraints and Market Power

Min Wang

2.1 Abstract

A recent literature of Green Paradox shows that green policy intended to alleviate the problem of climate change may turn to speed it up (Sinn, 2008). The goal of this paper is to examine the climate change impacts of renewable energy policies by focusing on capacity constraints of renewable energies and market power, both of which turn out to play important roles in determining policy impacts on the time profile of fossil fuel supply as well as the time pattern of greenhouse gas (GHG) emissions. By recognizing the capacity constraints, in this paper, we distinguish renewable energies between capacity constrained renewable energies and abundant renewable energies, and study their price policies and quantity policies in competitive market and non-competitive market. We show in this paper that the Green Paradox can only be confirmed in the benchmark case. Moreover, after considering the two factors mentioned, Green Paradox may not exist: the capacity constraints help renewable energy to delay the fossil fuel use to the distant future and the existence of market power changes the optimization rule of fossil fuel owners as well as their response to renewable energy policies.

2.2 Introduction

As a response to global climate change and high energy prices, major economies throughout the world are promoting the development of renewable energies such as biofuels, wind and solar energies. Government support for renewable energies takes many forms, including direct price subsidies, quantity mandates and R&D subsidies. The objective of this research is to evaluate the economic as well as greenhouse gas (GHG) impacts of government supports for renewable energies.

It may at first appear that government support can help substitute clean renewable energies for fossil fuels and therefore benefit the environment. However this conventional wisdom might not hold once owners of fossil fuels respond to renewable energy supplies. In a recent important paper, Sinn (2008) shows that policies reducing demand for fossil fuels, e.g. increasing tax rate on carbon, improving energy efficiency and increasing the use renewable energies, could lead to overextraction of fossil fuels in the near future and exacerbate the problem of climate change. Why? All these demand reducing policies depress the price of fossil fuels more in the future than at present, encouraging fossil fuel owners to increase present extraction. Sinn (2008) uses "Green Paradox" to describe this unintended consequence. Hoel (2009) and Van Ploeg and Withagen (2010) examine the "Green Paradox" effect of reducing the cost of backstop technology. Gronwald etc. (2010) extend Sinn's work by incorporating endogenous capacity adjustment cost for fossil fuels' extraction and show that "Green Paradox" may not exist. The main innovation of this paper is to recognize the capacity constraints of some renewable energies and the existence of market power in the fossil fuel sector, and evaluate how those two factors change the impacts of renewable energy policies. We show that the "Green Paradox" might not exist once the two factors are considered.

Traditionally renewable energies are modeled as "backstop" resources: once their costs of production are low enough, they will drive fossil fuels completely out of the market, e.g. Hoel (1978, 1983), Dasgupta and Stiglitz (1981), Dasgupta et al. (1982, 1983) and Chakravorty et al. (2006, 2008). However many renewable energies have capacity constraints. For instance, biofuels' capacity is limited by land availability and increasing demand for food and feed. Although second generation biofuel (e.g. cellulosic ethanol) has the promise of significantly increasing the capacity, it is not yet competitive with other fuel sources. Even when it becomes competitive, its capacity is still limited by the availability of arable land. Wind power is the fastest growing renewable energy source in the world, and its capacity in the US has increased dramatically in recent years, rising from 1600 megawatts in 1994 to more than 9200 megawatts in 2005 (Aabakken, 2006). However, as the capacity increases, prime wind sites are used up and less favorable sites will have to be used, increasing the siting cost. Further, these sites are usually located far away from consumption centers, leading to significant transportation costs and increased pressure on the electricity transmission grid. The cost is thus expected to increase sharply after a threshold, e.g., if wind energy replaces fossil fuels as the dominant energy source. Therefore, we can distinguish capacity constrained renewable energies, such as biofuels, wind and hydropower, from abundant renewable energies like solar. In all follows, we use biofuels and solar to represent these two categories of renewable energies. For the former one, we can in further divide them into low cost biofuels, which have been competitive on the market, e.g. sugarcane ethanol in Brazil, and high cost biofuels like second generation biofuels, whose price is not competitive yet.

We similarly classify the renewable energy polices into solar cost reduction policies, cost reduction policies for high cost biofuels and capacity expansion policies for high (low) cost biofuels.[1] The policies include direct price subsidy, such as roof plan in German, \$0.51 per gallon subsidy to biofuels in U.S. and R&D support. For instance, in the U.S., the federal R&D spending on biofuels has been between \$50 and \$100 million per year between 1978 and 1998, and the Biomass Research and Development Program offers \$12 million R&D support for bioenergy related research (Gielecki, Mayes and Prete, 2001). In the literature, Amigues at.al (1998) and Holland (2003) are the few papers that show the important role of capacity constraints of renewable resources. They find that capacity constraints could change the order of extraction of heterogeneous resources, for the scarcity rent generated by the capacity constraints changes the cost order of different resources. Similarly, by considering the solar cost reduction policies in the competitive market and comparing its impacts to other policies, we find that the capacity constraints facing biofuels play an important role in determining the policy impact on the time profile of fossil fuel use as well as the time pattern of GHG emissions.

The "Green Paradox" literature commonly assumes competitive energy markets. But market power exists in many energy markets, especially in the oil sector. In addition to OPEC, in many nations, such as Russia, China and Venezuela, oil supply is monopolized by state owned enterprises. In this paper, we study both the case of competitive energy market and the case of non-competitive energy market, where a monopolist controls the supply of fossil fuels. Even when "Green Paradox" arises in a competitive market, it may not arise under monopoly: in response to renewable energy policies, a monopolist might reduce fossil fuel extraction in the short run.

This paper also contributes to the debate on the carbon footprint of biofuels. Most life cycle analysis of biofuels ignores the dynamic, long-run response of fossil fuel supply to biofuel development. We show that biofuels are possible to help delay GHG emissions of fossil fuels to the future, helping mitigate GHG emissions.

The rest of the paper is organized as follows. Section 2 analyzes the impacts of

[1] Since low cost biofuels have been competitive and supplied the market at full capacity from the beginning, further reducing their price would not have any impact on the equilibrium energy price and quantities. Therefore for low cost biofuels, we only consider the capacity expansion policies.

renewable energies on the fossil fuel use and the climate change in a competitive energy market. The case of non-competitive market is examined in section 3. Section 4 concludes this paper. Additional figures and definition of the fossil fuel stock are in the Appendix A. Proofs of major results are in the Appendix C.

2.3 Competitive fossil fuel market

We consider a partial equilibrium model where policies are exogenously given. We assume that the four energy products, fossil fuels, low cost biofuels, high cost biofuels and solar, are perfect substitutes.[2] In this section, we assume all sectors of renewable energies are perfectly competitive and leave the case of non-competitive market to the next section.

Let $q_f(t)$ and $X(t)$ be the supply and remaining reserve of fossil fuels in period t with the starting reserve X_0. Denote $q_{b,i}(t)$, $i = \{l, h\}$, the output of biofuels in period t and $\overline{q}_{b,i}$, $i = \{l, h\}$, the capacity constraints of production where l and h represent low cost biofuels and high cost biofuels respectively. Thus $q_{b,i}(t) \leq \overline{q}_{b,i}$ for all t. In addition, we let $q_b(t)$ and $\overline{q}_b$ denote the total supply $q_{b,l}(t) + q_{b,h}(t)$ and total capacity $\overline{q}_{b,l} + \overline{q}_{b,h}$ of these two kinds of biofuels. Suppose all energies have constant marginal cost. Let the unit production cost of fossil fuels, biofuels and solar be c_f, $c_{b,l}$, $c_{b,h}$ and c_s, ordered as $c_f < c_{b,l} < c_{b,h} < c_s$. Financed by general taxation, the government can reduce $c_{b,i}$ and c_s, and increase $\overline{q}_{b,i}$, $i = \{l, h\}$.

We assume a stable energy demand function $p = h(Q)$ with $h'(Q) \leq 0$ and $h''(Q) \geq 0$, where p denotes the energy price and Q is the total demand. In addition, we assume $\overline{q}_{b,h} + \overline{q}_{b,l} < h^{-1}(c_s)$ such that total production of biofuels can not satisfy market demand when solar becomes competitive on the market. Finally X_0 is assumed to be sufficiently large such that initial energy price $p(0)$ is less than marginal cost of high cost biofuels, but higher than marginal cost of low cost biofuels. Given this assumption, low cost biofuels is supplied at full capacity from the beginning, i.e. $t = 0$. In the Appendix B, we define the critical values of fossil fuel stock which guarantee $c_{b,l} < p(0) < c_{b,h}$.

[2] The main end uses of energy are electricity and transportation. The substitution is possible with adjustment of energy conversion to different end uses. For example, oil can be converted to transportation through gasoline and diesel. Solar can be converted to electricity through photovoltaic technology and then can be used for transportation through electricity car. More details about the conversion can see Chakravorty et.al (2008). They also compute the cost of converting energy resources into different end uses.

2.3.1 The energy market

Since the renewable energy sector is perfectly competitive and marginal production cost is constant, the optimal supply of biofuels is

$$q_{b,i}(t)\begin{cases} = 0, & \text{if } p(t) < c_{b,i} \\ \in \left[0, \overline{q}_{b,i}\right], & \text{if } p(t) = c_{b,i} \\ = \overline{q}_{b,i}, & \text{if } p(t) > c_{b,i} \end{cases}, \tag{2.1}$$

$i = \{l, h\}$. Similarly, given the supply of the other two energies, the optimal supply of solar is

$$q_s(t)\begin{cases} = 0, & \text{if } p(t) < c_s \\ \in \left[0, h^{-1}(c_s)\right], & \text{if } p(t) = c_s \\ h^{-1}(p(t)), & \text{if } p(t) > c_s \end{cases} \tag{2.2}$$

Since solar can supply the whole market when $p(t) = c_s$, it is impossible for the energy price to exceed c_s. As in the literature, the unit cost of solar c_s plays the role of price ceiling in the energy market.

Given the market price, fossil fuel supply is determined by the optimization problem

$$\underset{\{q_f(t)\}}{Max} \int_0^\infty e^{-rt}\left[p(t)\, q_f(t) - c_f q_f(t)\right] dt$$
$$s.t. \dot{X}(t) = -q_f(t); \int_0^\infty q_f(t)\, dt = X_0;$$

where r is the market interest rate. Let λ be the present shadow value of fossil fuel stock. Then by solving the problem, the fossil fuel supply follows the Hotelling rule that the market price is equal to current shadow value of fossil fuel stock plus the unit extraction cost of fossil fuels

$$h(q_f(t) + q_b(t) + q_s(t)) = c_f + \lambda e^{rt} \tag{2.3}$$

If fossil fuel owners extract fossil fuels and invest them in the capital market, they can earn the return of interest rate every period. The shadow value λe^{rt} measures that opportunity cost of leaving resource in the underground. Following Holland (2003), we define the right hand side of (2.3) as augmented marginal cost ($AMC(t)$). Note that by (2.3), energy price must continuously increase over time as long as the stock of fossil fuels remains.

Now we define the market equilibrium as follows: the market equilibrium is the sequence of $\{q_f(t), q_{b,l}(t), q_{b,h}(t), q_s(t), p(t)\}_{t=0}^{\infty}$ such that (a) given the sequence of energy prices, $\{q_f(t), q_{b,l}(t), q_{b,h}(t), q_s(t)\}_{t=0}^{\infty}$ solve the optimal supply of fossil fuels and renewable energies by (2.1), (2.2) and (2.3); (b) the energy market is clear $p(t) = h(q_f(t) + q_{b,h}(t) + q_{b,l}(t) + q_s(t))$.

By the supply rules (2.1), (2.2) and (2.3), the supply of fossil fuels experiences the following three stages:

During the first stage $[0, T_1]$, the energy price $p(t)$ is less than the marginal cost of high cost biofuels $c_{b,h}$. Since low cost biofuels supply the market at its full capacity from the beginning, the fossil fuels supply the residual market as

$$q_f(t) = h^{-1}\left(\lambda e^{rt} + c_f\right) - \overline{q}_{b,l} \tag{2.4}$$

During this stage, the market price keeps increasing according to (2.3) until T_1 when the price equals to $c_{b,h}$.

During stage two, $[T_1, T]$, both the fossil fuels and the two kinds of biofuels are produced. Since $p(T_1) = c_{b,h}$, high cost biofuels become competitive at T_1 and its supply jumps up from zero to $\overline{q}_{b,h}$ and supply of fossil fuels jumps down by $\overline{q}_{b,h}$ to guarantee price continuity. After T_1, energy price keeps increasing until it reaches c_s at time T, $p(T) = c_s$. In this stage, the supply of fossil fuels follows

$$q_f(t) = h^{-1}\left(\lambda e^{rt} + c_f\right) - \overline{q}_b \tag{2.5}$$

Since the price stops increasing when solar becomes competitive, it is optimal for fossil fuels owner to exhaust the reserve before the arrival time of solar. The extraction of fossil fuels thus stops or jumps to zero at time T. For high cost biofuels, they are supplied at full capacity from T_1 on.

During the third stage, $[T, \infty]$, fossil fuel stock is exhausted and solar becomes competitive and begins to supply the market. In this stage, energy price keeps constant at c_s, $q_f(t) = 0$, $q_{b,i}(t) = \overline{q}_{b,i}$, $i = \{l, h\}$, and solar supplies the rest demand $h^{-1}(c_s) - \overline{q}_b$.

We illustrate the above price path and supply path of fossil fuels in Figure 2.1.

Finally we need to solve $\{\lambda, T_1, T\}$ to characterize the equilibrium path. Given $\overline{q}_{b,i}$, $i = \{l, h\}$, and c_j, $j = \{f, b, s\}$, the solutions of $\{\lambda, T_1, T\}$ are determined by

$$\int_0^{T_1} h^{-1}\left(\lambda e^{rt} + c_f\right) dt + \int_{T_1}^{T} \left[h^{-1}\left(\lambda e^{rt} + c_f\right) - \overline{q}_{b,h}\right] dt - T\overline{q}_{b,l} = X_0 \tag{2.6}$$

$$c_f + \lambda e^{rT_1} = c_{b,h} \tag{2.7}$$

$$c_f + \lambda e^{rT} = c_s \tag{2.8}$$

(2.6) is the resource exhaustion condition, i.e. the total extraction of fossil fuels (left hand side of (2.6)) should be equal to total reserve of fossil fuels (right hand side of (2.6)). (2.7) and (2.8) are the Hotelling rule at time T_1 and T.

2.3.2 Policy impacts

Given (2.6) — (2.8), we can obtain the impacts of renewable energy policies by a simple exercise of comparative statics.

Proposition 1. *Suppose energy market is perfectly competitive,*

(1) solar cost reduction policies reduce the present shadow value of fossil fuels, delay the arrival time of high cost biofuels and bring forward the exhaustion time of fossil fuels, i.e. $\partial\lambda/\partial c_s > 0$, $\partial T_1/\partial c_s < 0$ *and* $\partial T/\partial c_s > 0$.

(2) cost reduction policies for high cost biofuels reduce the present shadow value of fossil fuels, bring forward the arrival time of biofuels and delay the exhaustion time of fossil fuels, i.e. $\partial\lambda/\partial c_{b,h} > 0$, $\partial T_1/\partial c_{b,h} > 0$ *and* $\partial T/\partial c_{b,h} < 0$.

(3) capacity expansion policies for both high cost and low cost biofuels reduce the present shadow value of fossil fuels, delay the arrival time of biofuels and the exhaustion time of fossil fuels, i.e. $\partial\lambda/\partial\overline{q}_{b,i} < 0$, $\partial T_1/\partial\overline{q}_{b,i} > 0$ *and* $\partial T/\partial\overline{q}_{b,i} > 0$, $i = \{l, h\}$.

Appendix C contains the proof of the Proposition 1. The policy impacts on price path, illustrated in Figure 2.1, 2.3 and 2.5, follows directly from Proposition 1.[3] Obviously all policies reduce shadow value of fossil fuels and thus lower down the energy price for all periods. Since all energies are substitutes, owners of fossil fuels are facing the residual demand. And as exhibited in Figures in Appendix A, if the energy market is competitive, all policies lower down the residual demand for fossil fuels. Moreover except capacity expansion policies for low cost biofuels, all other renewable energy policies only reduce future residual demand for fossil fuels. That means the future price for fossil fuels would fall under these policies, which provides an incentive for fossil fuel owners to increase current supply. As the consequence, the current price as well as shadow value of fossil fuels drops. As for the capacity expansion policies for low cost biofuels, since they increase the current energy supply on the market, they directly reduce the current price and shadow value of fossil fuels.[4]

[3] The bold dashed line represents the path after policy implementation.

[4] Since capacity expansion policies for low cost biofuels reduce both present and future residual demand for fossil fuels, fossil fuel owners may respond by reducing their current supply as shown below. But the change of present supply of fossil fuels is the indirect effect of increased capacity or supply of low biofuels and generically the direct effect would dominate the indirect effect. Therefore the present total supply of energy would increase under those policies even when fossil fuel supply falls.

Following Figure 2.1, 2.3 and 2.5., we derive the corresponding policy impacts on fossil fuel use in Figure 2.2, 2.4 and 2.6. As capacity expansion policies for low cost biofuels, it is hard to obtain their impacts for general demand function. But if the demand is linear, we have (see Appendix C for proof)

Corollary 2. *Suppose energy market is perfectly competitive and demand is linear, capacity expansion policies for low cost biofuels at least reduce fossil fuel use in early stages.*

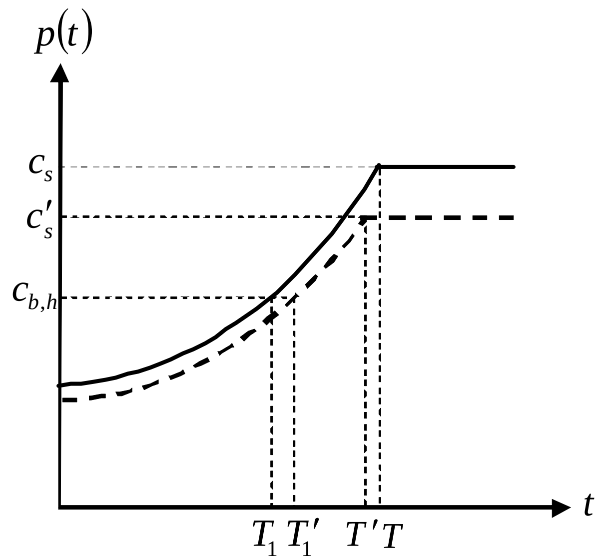

FIGURE 2.1. Solar cost reduction policies: impact on price

Despite same price impacts, impacts of these renewable energy policies on fossil fuel use are diversified: solar cost reduction policies increase fossil fuel use for all periods, the two renewable energy policies for high cost biofuels increase current extraction of fossil fuels and capacity expansion policies for low cost biofuels can depress the present extraction of fossil fuels. Since they depress the residual demand for fossil fuels from the beginning, capacity expansion policies for low cost biofuels generate two countervailing effects on fossil fuel supply: the depressed future (current) price encourages fossil fuel owners to increase (decrease) the present fossil fuel supply. That explains why they could reduce present fossil fuel supply.

Now we go on to discuss how capacity constraints of renewable energies make difference in policy impacts on fossil fuel use. Firstly consider solar energy, since it is not capacity constrained and capable to supply the whole market when it becomes economic

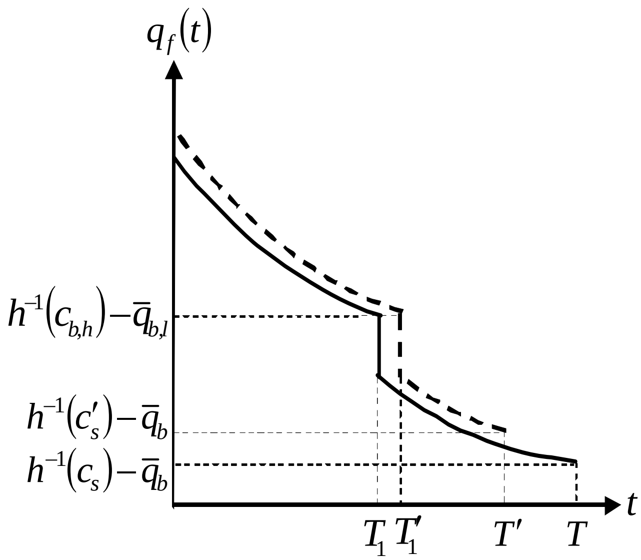

FIGURE 2.2. Solar cost reduction policies: impact on fossil fuel use

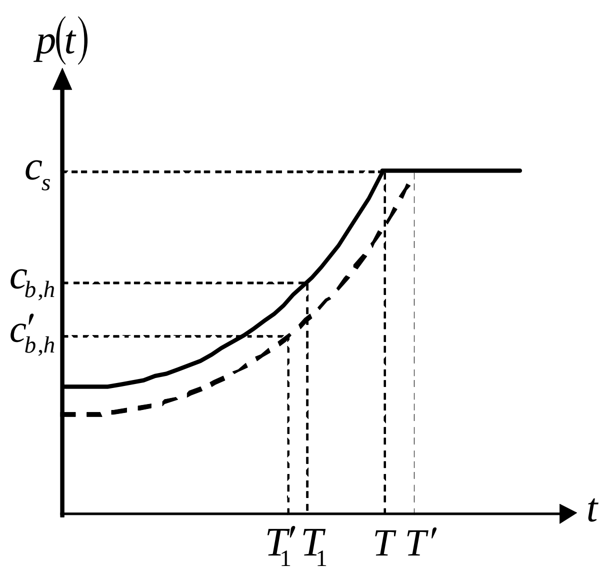

FIGURE 2.3. Cost reduction policies for high cost biofuels: impact on price

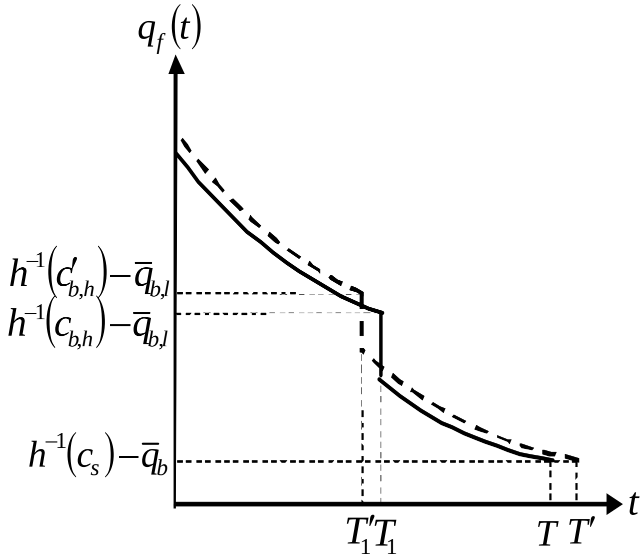

FIGURE 2.4. Cost reduction policies for high cost biofuels: impact on fossil fuel use

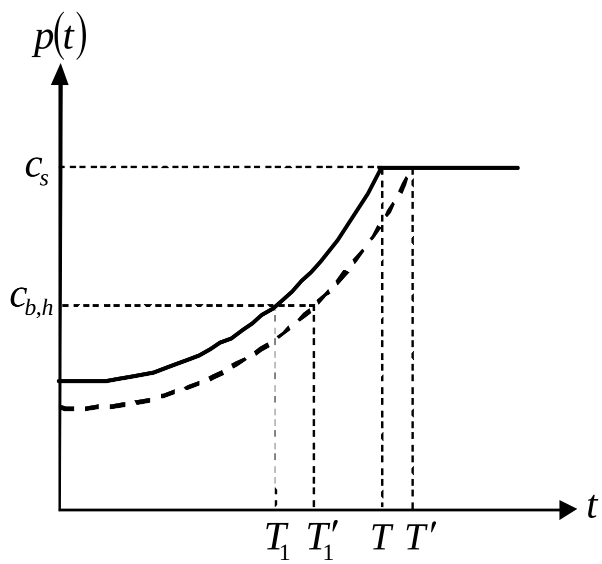

FIGURE 2.5. Capacity expansion policies for biofuels: imapct on price

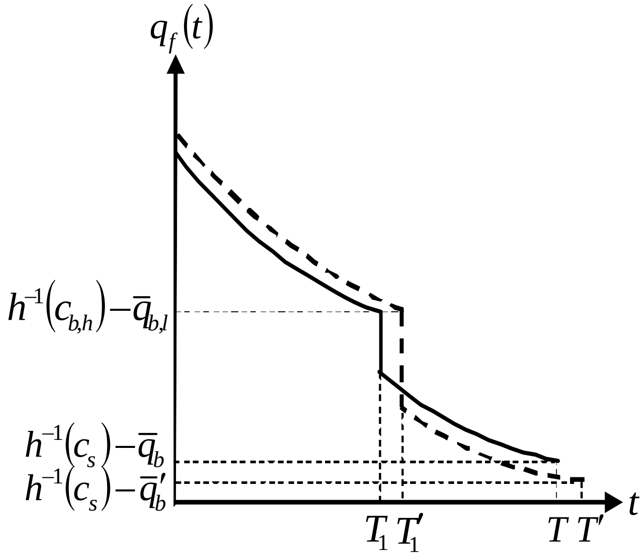

FIGURE 2.6. Capacity expansion policies for high cost biofuels: impact on fossil fuel use

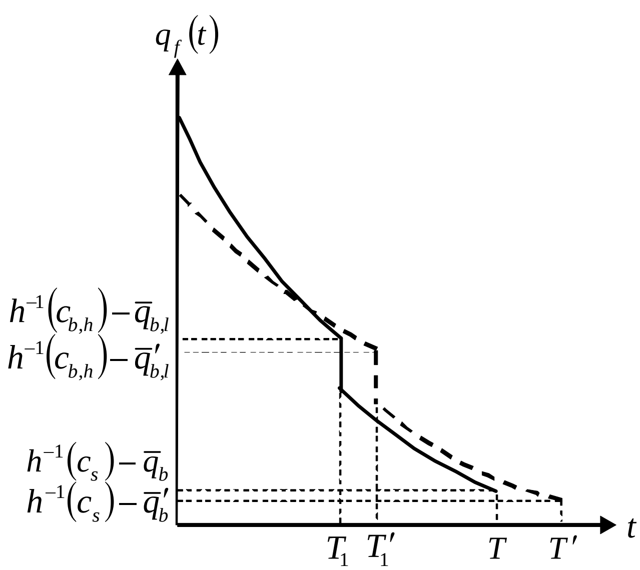

FIGURE 2.7. Capacity expansion policies for low cost biofuels: impact on fossil fuel use

to use, fossil fuels have to be exhausted as energy price hits solar's marginal cost. As solar cost drops, fossil fuel owners have a shorter time to exhaust the fossil fuels and have to expediate the extraction of fossil fuels in all periods as Figure 2.2 shows. In contrast, after they become competitive, biofuels, owing to their capacity constraints, supply the market simultaneously with fossil fuels. That implies biofuels can substitute and delay the fossil fuel use to the distant future after they become competitive. Therefore by enhancing substitution effect of biofuels, capacity expansion policies reduce the fossil fuel use after they become competitive and postpone the exhaustion time of fossil fuels. Due to their different time of availability, capacity expansion policies for low cost biofuels can reduce fossil fuel use from now on and capacity expansion policies for high cost biofuels can only realize that after T_1. As cost reduction policies for high cost biofuels, their only beneficial effect is to bring forward the arrival time of biofuels. Fossil fuel use falls for a while during that advanced time span $[T_1, T_1']$ but rises in all other periods due to reduced shadow value of fossil fuels. We show the former effect dominates the latter one and cost reduction policies for high cost biofuels also help postpone the exhaustion time of fossil fuels.

Since GHGs are generated by fossil fuel use and are accumulating over time, the previously discussed policy impacts on fossil fuel use would finally affect the total damage of GHGs, which is evaluated by summing up discounted damage of GHGs in all periods. By aforediscussed policy impacts, we notice two impacts play the roles in climate change: the change of fossil fuel use in early stages and the change of exhaustion time of fossil fuels. The first effect playing the role owns to the discount factor, which cause the total damage of GHGs more sensitive to early damage.[5] The second effect, exhaustion time of fossil fuels, measures the average extraction rate of fossil fuels. Long exhaustion time means, on average, both the rate of fossil fuel use and accumulation rate of GHG emission are slow, resulting in a smaller damage of GHGs. In the literature, previous works define "Green Paradox" either as increasing the present fossil fuel use or as bringing forward the exhaustion time. Here we define a strong version of "Green Paradox" that a renewable energy policy has "Green Paradox" if and only if it simultaneously generates

[5] Actually due to the convexity of damage function of GHGs, delaying one unit of current emission to the distant future increases that unit's marginal damage. Here we implicitly assume that the discounted value of that higher future marginal damage is less than current marginal damage. In the literature of Green Paradox, delaying today's fossil fuels use to the distant future is considered as being beneficial to the environment, see Sinn (2008) for an example. Hoel (2010) derives a general condition under which delaying current emission to the future would be beneficial and argues the condition is easily to be satisfied.

those two consequences. Evidently by increasing the fossil fuel use in all periods and bringing forward the exhaustion time of fossil fuels, solar cost reduction polices generate the "Green Paradox" and aggravate the problem of global climate change. Capacity expansion policies for low cost biofuels, in contrast, can lead to the opposite direction, benefiting the environment. By increasing fossil fuel use in early stages and delaying the exhaustion time of fossil fuels, impacts of other two policies on climate change are ambiguous. We collect these results in Proposition 3 below

Proposition 3. *Suppose energy market is perfectly competitive, the "Green Paradox"*

(1) can only be confirmed in the benchmark case – solar cost reduction policies;

(2) does not exist for capacity expansion policies for low cost biofuels;

(3) is ambiguous for the two high cost biofuel policies.

No matter what final impact of the two high cost biofuels policies would be, an important lesson we can learn so far is that, after considering the response of fossil fuel use, renewable energy policies may be not as expected to alleviate the problem of global climate change. Instead, they are possible to reinforce the problem. More important, the capacity constraints of renewable energies play an important role in determining policy impacts and hence should not be ignored in policy analysis.

2.4 Non-competitive fossil fuel market

In this section, we consider the situation where the fossil fuel sector is not competitive. In particular, to sharpen our results, we assume that the sector is controlled by a monopolist. The monopolist owns entire stock of exhaustible fossil fuels X_0. As previous section, we study the impacts of the four types of policies on the price path and supply path of fossil fuels first and then evaluate their impacts on climate change.

2.4.1 The energy market

Let T be the exhaustion time of fossil fuels. The monopolist's payoff is the discounted value of profit over the period $[0, T]$. Given the supply function (2.1) and (2.2), the monopolist's problem is

$$\underset{\{q_f(t)\}}{Max} \int_0^T e^{-rt} \left[h\left(q_f\left(t\right) + q_{b,h}\left(t\right) + q_{b,l}\left(t\right) + q_s\left(t\right)\right) q_f\left(t\right) - c_f q_f\left(t\right)\right] dt$$

$$s.t. \dot{X}\left(t\right) = -q_f\left(t\right); \int_0^T q_f\left(t\right) dt = X_0;$$

We assume the revenue function $h(q_f + q_b + q_s) q_f$ is concave in q_f.

Let μ be the present shadow value of fossil fuel stock. Then the Hamiltonian at time t can be written as

$$H_t = h(q_f(t) + q_b(t) + q_s(t)) q_f(t) - c_f q_f(t) - \mu e^{rt} q_f(t) \tag{2.9}$$

The optimal condition is

$$h'(q_f(t) + q_b(t) + q_s(t)) q_f(t) + h(q_f(t) + q_b(t) + q_s(t)) = c_f + \mu e^{rt} \tag{2.10}$$

(2.10) implies that if $q_f(t)$ has interior solution, marginal revenue of monopolist should be equal to the augmented marginal cost of fossil fuels. Finally free choice of exhaustion time T leads to the transversality condition

$$H_T = h(q_f(T) + q_b(T) + q_s(T)) q_f(T) - c_f q_f(T) - \mu e^{rT} q_f(T) = 0 \tag{2.11}$$

Since at the exhaustion time T, $p(T) = c_s$, equation (2.11) implies

$$c_s = c_f + \mu e^{rT} \tag{2.12}$$

The market equilibrium is the sequence of $\{q_f(t), q_{b,l}(t), q_{b,h}(t), q_s(t), p(t)\}_{t=0}^{\infty}$ such that (a) given the sequence of energy prices, $\{q_f(t), q_{b,l}(t), q_{b,h}(t), q_s(t)\}_{t=0}^{\infty}$ solve the optimal supply of fossil fuels and renewable energies by (2.1), (2.2) and (2.10); (b) the energy market is clear $p(t) = h(q_f(t) + q_{b,h}(t) + q_{b,l}(t) + q_s(t))$.

We go on to characterize the price path and supply path of fossil fuels.

Firstly we need to determine the marginal revenue, which is discontinuous due to the constraints of (2.1) and (2.2). Figure 2.8 exhibits the marginal revenue for the monopolist and presents the three output levels of fossil fuels, at which the marginal revenue jumps. When the monopolist chooses the output level $q_f(t) = h^{-1}(c_{b,h}) - \overline{q}_{b,l}$ or $h^{-1}(c_s) - \overline{q}_b$, the corresponding market prices is $c_{b,h}$ or c_s, at which high cost biofuels or solar become competitive. At those two output levels, if the monopolist reduces its own production, the high cost biofuels or solar will fill in the market at the price $c_{b,h}$ or c_s, sustaining a constant price. That means, at $q_f(t) = h^{-1}(c_{b,h}) - \overline{q}_{b,l}$ and $h^{-1}(c_s) - \overline{q}_b$, $h'(\cdot)$ is discontinuous with $h'_-(\cdot) = 0$ and $h'_+(\cdot) < 0$. Therefore marginal revenue jumps up at those two output level and for $q_f(t) \in \left[h^{-1}(c_{b,h}) - \overline{q}_{b,l},\ h^{-1}(c_{b,h}) - \overline{q}_b\right]$ and $q_f(t) \in [0,\ h^{-1}(c_s) - \overline{q}_b]$, marginal revenue keeps constant at $c_{b,h}$ and c_s. In those two output ranges, the competition from

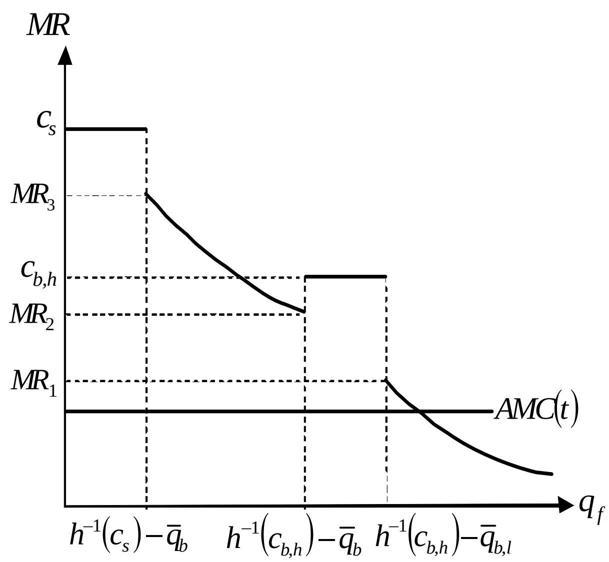

FIGURE 2.8. Marginal revenue for the monopolist

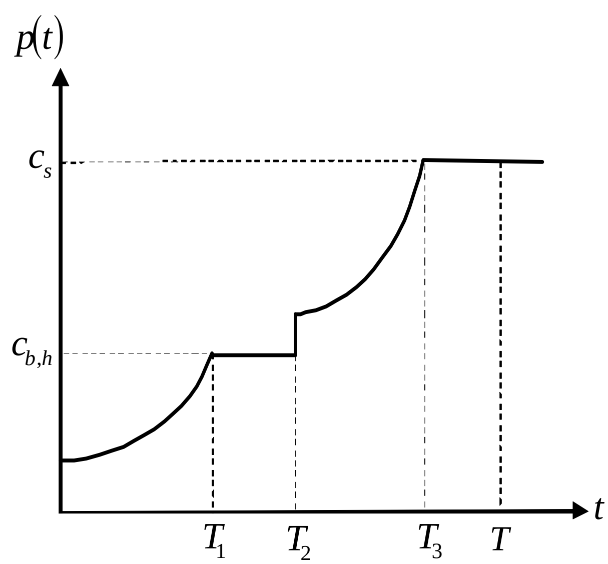

FIGURE 2.9. Price path in non-competitive market

biofuels and solar constrains the market power of monopolist such that $h'(\cdot)=0$. When $q_f(t)$ is reduced to $h^{-1}(c_{b,h})-\overline{q}_b$, high cost biofuels cannot supply the market in further due to the capacity constraints and the monopolist regains the market power, leading $h'(\cdot)$ jumps from $h'(\cdot)=0$ to $h'(\cdot)<0$. Correspondingly, marginal revenue jumps down. Note that marginal revenue does not exist between $[MR_1,\, MR_2]$ and $[MR_3,\, c_s]$.[6]

Now based on (2.10) and Figure 2.8, we can determine the path of energy price and fossil fuel use. Figure 2.9 illustrates the corresponding price path in the energy market.

First we should be aware that the solution of fossil fuel use is determined by how $AMC(t)$ intersects with marginal revenue in Figure 2.8. And as shown in Figure 2.8, $AMC(t)$ is a horizontal line and continuously increases over time. But marginal revenue is not continuous. That generates possible corner solution and interior solution for fossil fuel use in differnet periods. In the following, we base the type of the solution of fossil fuel use to divide the stage of price path and supply path of fossil fuels.

During the first stage $[0,T_1]$, fossil fuel supply has interior solution determined by equating marginal revenue to $AMC(t)$

$$h'\left(q_f(t)+\overline{q}_{b,l}\right)q_f(t)+h\left(q_f(t)+\overline{q}_{b,l}\right)=c_f+\mu e^{rt} \tag{2.13}$$

until T_1 when the energy price rises to $c_{b,h}$ and marginal revenue rises to MR_1. In this stage, low cost biofuels are supplied at full capacity from the beginning and fossil fuels supply the residual demand at a decreasing rate. Supply of high cost biofuels and solar is equal to zero.

Stage two is $[T_1,T_2]$. Marginal revenue jumps at T_1 but $AMC(t)$ still continuously increases. Therefore after T_1, the marginal revenue $c_{b,h}$ would be higher than $AMC(t)$ for a while when $AMC(t)\in[MR_1,MR_2]$, leading to corner solution of fossil fuel supply, i.e., monopolist would flood the market by staving off high cost biofuels.[7]

$$q_f(t)=h^{-1}(c_{b,h})-\overline{q}_{b,l}. \tag{2.14}$$

Note that as the monopolist takes the corner solution of (2.14), the market price keeps flat at $c_{b,h}$. That is optimal for the monopolist because if the monopolist let the price continuously increase at T_1, it has to reduce the fossil fuel supply dramatically, leading

[6] The three critical marginal revenues are defined as $MR_1{=}h'\left(h^{-1}(c_{b,h})\right)\left[h^{-1}(c_{b,h})-\overline{q}_{b,l}\right]+c_{b,h}$; $MR_2{=}h'\left(h^{-1}(c_{b,h})\right)\left[h^{-1}(c_{b,h})-\overline{q}_b\right]+c_{b,h}$; $MR_3{=}h'\left(h^{-1}(c_{b,s})\right)\left[h^{-1}(c_{b,s})-\overline{q}_b\right]+c_s$.

[7] During $[T_1,T_2]$, the domain of $q_{f,t}$ is $\left[0,h^{-1}(c_{b,h})\right]$. Therefore as marginal revenue is greater than augmented marginal cost, the corner solution is $q_{f,t}=h^{-1}(c_{b,h})-\overline{q}_{b,l}$.

to a plummet of profit.[8] Then when will the monopolist stop staving off high cost biofuels and let the price increase? As $AMC(t)$ increases in further such that $AMC(t) \in [MR_2, c_{b,h}]$, there exists two solutions for $q_f(t)$: one is the corner solution as before and the other is interior solution determined by equating marginal revenue to $AMC(t)$. Let T_{MR_2} and $T_{c_{b,h}}$ be the time such that $AMC(T_{MR_2}) = MR_2$ and $AMC\left(T_{c_{b,h}}\right) = c_{b,h}$. If the monopolist takes corner solution until $T_{c_{b,h}}$, its profit would decrease to zero at $T_{c_{b,h}}$. That cannot be the optimum. Therefore the monopolist must stop staving off high cost biofuels in some period before $T_{c_{b,h}}$. Let T_2 be that period and we determine T_2 as follows (see Appendix C for proof)

Proposition 4. *Under monopoly, when the energy price rises to cost of high cost biofuels, the monopolist will stave off high cost biofuels until T_2, in which the economic value for interior solution of fossil fuel supply is equal to that for corner solution of fossil fuel supply, i.e.*

$$q_f(T_2)\left[h\left(q_f(T_2)+\overline{q}_b\right)-c_f-\mu e^{rT_2}\right]=\left[h^{-1}(c_{b,h})-\overline{q}_{b,l}\right]\left(c_{b,h}-c_f-\mu e^{rT_2}\right) \qquad (2.15)$$

where $q_f(T_2)$ is the residual demand for fossil fuels in T_2.

The result of Proposition 4 is straightforward if we look into the path of Hamiltonian $H(t)$ in Figure 2.10. During $\left[T_{MR_2}, T_{c_{b,h}}\right]$, $H(t)$ has two paths corresponding to two possible solutions of fossil fuel use. The optimal solution implies that if in some period, there are two possible solutions for fossil fuel supply, the monopolist should choose the one that generates larger economic value, i.e. $H(t)$. That ensures the monopolist to gain maximal economic profit in every period, which eventually leads to optimization of lifetime economic profit. The optimal solution of T_2 also guarantees the continuity of economic value over time. But energy price has to jump up at T_2 as shown in Figure 2.9. This study is the first one examining the path of resource price when backstop technology is capacity constrained and to our knowledge, the discontinuous price path for a resource has not been studied so far in the literature of resource economics.

We summarize the above results in the following Proposition,

Proposition 5. *As the energy price rises to the cost of high cost biofuels at T_1, the monopolist would stave off high cost biofuels for a while by keeping energy price flat at $c_{b,h}$ and flooding the market. Eventually, in period T_2, the monopolist reduces production*

[8] As will be shown below, in optimum, the profit for the monopolist should be continuous over time.

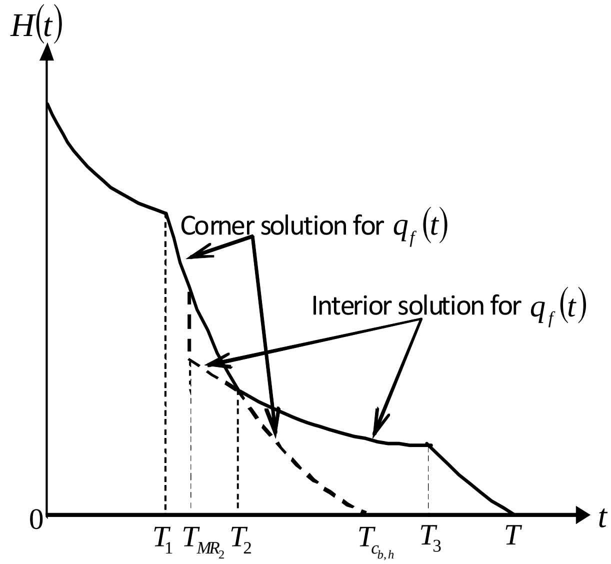

FIGURE 2.10. Path of Hamiltonian

dramatically and let backstop produce at full capacity. Then the energy price jumps above the cost of high cost biofuels in T_2*.*

Stage three $[T_2, T_3]$ starts when high cost biofuels begin to supply at full capacity and the optimal supply rule of fossil fuels is again determined by equating the marginal revenue to augmented marginal cost

$$h'\left(q_f\left(t\right)+\overline{q}_b\right)q_f\left(t\right)+h\left(q_f\left(t\right)+\overline{q}_b\right)=c_f+\mu e^{rt} \tag{2.16}$$

As in the first stage, energy price keeps rising with decrease of fossil fuel use. At the end of this stage T_3, energy price reaches c_s and the supply of fossil fuels falls to $h^{-1}(c_s)-\overline{q}_b$.

During stage four $[T_3, T]$, energy price is equal to c_s and supply of solar becomes competitive for the energy market. As in the second stage, the marginal revenue of monopolist jumps at T_3 and the optimal supply of fossil fuels has corner solution

$$q_f\left(t\right)=h^{-1}\left(c_s\right)-\overline{q}_b \tag{2.17}$$

i.e. monopolist floods the market to stave off solar during the whole stage until the reserve is exhausted. Intuitively, since energy price cannot exceed c_s, any delay of fossil fuel extraction would incur the interest cost without any benefit. Therefore, at this stage monopolist would extract its stock of fossil fuels as soon as possible. By (2.10) and (2.12), we can know that $T - T_3$ is pre-determined

$$T - T_3 = \frac{1}{r} \ln \left(\frac{c_s - c_f}{h' \left(h^{-1}(c_s)\right) \left(h^{-1}(c_s) - \overline{q}_b\right) + c_s - c_f} \right) \tag{2.18}$$

Therefore the stock $X(T_3)$ remains at T_3 is also exogenously given and does not depend on stock of fossil fuels

$$X(T_3) = \frac{h^{-1}(c_s) - \overline{q}_b}{r} \ln \left(\frac{c_s - c_f}{h' \left(h^{-1}(c_s)\right) \left(h^{-1}(c_s) - \overline{q}_b\right) + c_s - c_f} \right) \tag{2.19}$$

It is straightforward to show $\partial X(T_3) / \partial \overline{q}_b < 0$ and $\partial (T - T_3) / \partial \overline{q}_b < 0$.

$[T, \infty]$ is the final stage, in which market energy price keeps constant at c_s, $q_f(t) = 0$, $q_{b,l}(t) = \overline{q}_{b,l}$, $q_{b,h}(t) = \overline{q}_{b,h}$ and solar supplies the rest of the demand $h^{-1}(c_s) - \overline{q}_b$.

Finally we need to solve $\{\mu, T_1, T_2, T_3, T, q_f(T_2)\}$ to characterize the equilibrium path. Given $\overline{q}_{b,l}$, $\overline{q}_{b,h}$ and c_i, $i = \{f, b, s\}$, the solutions of $\{\mu, T_1, T_2, T_3, T, q_f(T_2)\}$ are determined by (2.12), (2.15) and

$$\begin{aligned}
\int_0^{T_1} q_f(t)\, dt + \left[h^{-1}(c_{b,h}) - \overline{q}_{b,l}\right](T_2 - T_1) + \int_{T_2}^{T_3} q_f(t)\, dt + X(T_3) &= X_0 && (2.20)\\
h'\left(h^{-1}(c_{b,h})\right)\left[h^{-1}(c_{b,h}) - \overline{q}_{b,l}\right] + c_{b,h} - c_f - \mu e^{rT_1} &= 0 && (2.21)\\
h'\left(q_f(T_2) + \overline{q}_b\right) q_f(T_2) + h\left(q_f(T_2) + \overline{q}_b\right) - c_f - \mu e^{rT_2} &= 0 && (2.22)\\
h'\left(h^{-1}(c_s)\right)\left[h^{-1}(c_s) - \overline{q}_b\right] + c_s - c_f - \mu e^{rT_3} &= 0 && (2.23)
\end{aligned}$$

(2.20) is the resource exhaustion condition for fossil fuels. The corresponding fossil fuel supply, $q_f(t)$, in (2.20) is derived from (2.13) and (2.16). (2.21), (2.22) and (2.23) respectively present the optimal condition at time T_1, T_2 and T_3, i.e. the marginal revenue of monopolist at those three periods should be equal to the corresponding augmented marginal cost.

2.4.2 Policy impacts

Given the path of fossil fuel supply determined above, we now can analyze how the afore-discussed policies affect the energy market. In the following, we respectively consider the impacts of solar cost reduction policies, cost reduction policies for high cost biofuels, capacity expansion policies for high cost biofuels and low cost biofuels.

First, consider the renewable energy policies that reduce the cost of solar, we can conclude that (see Appendix C for the proof)

Proposition 6. *Under monopoly, solar cost reduction policies*

(1) increase the present shadow value of fossil fuels, i.e. $\partial\mu/\partial c_s < 0$*;*

(2) bring forward $\{T_1, T_2, T_3, T\}$*, i.e.* $\partial T_i/\partial c_s > 0$*,* $i = \{1, 2, 3\}$*, and* $\partial T/\partial c_s > 0$*.*

Figure 2.11 illustrates the conclusions of Proposition 6. Since present value of fossil fuels determines the equilibrium path of $\{q_f(t), q_{b,h}(t), p(t)\}_{t=0}^{\infty}$ as well as $\{T_1, T_2, T_3, T\}$, the result of Proposition 6.(1) is critical. If government implements policies reducing the cost of solar, the monopolist would respond by increasing the current price of fossil fuels and thus reducing the present supply of fossil fuels. The result seems surprising, for traditional convention would expect the price of fossil fuels drops when its future substitute good becomes cheaper. That is exactly what occurs in the case of competitive market. Recall that takes place in the case of competitive market because fossil fuel owners base their optimal decision on price and solar cost reduction policies depress their future price. However, when optimizing profit over time, monopolist is looking at the marginal revenue instead of market price of fossil fuels. And the marginal revenue, as shown in Figures in Appendix A, is depressed in some range, but in the meantime raised in some other range by all renewable energy policies considered. Therefore there exist two countervailing effects for renewable energy policies: depressed (increased) future marginal revenue induces monopolist to move future's (today's) supply to today (future). As for solar cost reduction policies, we show that, for any demand function, the effect of increased future marginal damage is in dominance. Hence solar cost reduction policies provide an incentive for the monopolist to delay today's fossil fuels supply to the future. Finally the current energy price or shadow value of fossil fuels rises. In an energy market without biofuels, Hoel (1978) obtains the same result for a special constant elasticity demand function. Proposition 6.(1) generalizes the finding of Hoel (1978) by considering a more general energy market and general demand function.

Following Proposition 6, we demonstrate the policy impacts on fossil fuel use in Figure 2.12. Although solar cost reduction policies turn to reduce the early use of fossil fuel, the stock of fossil fuels is exhausted in a shorter time as in the case of competitive market. The aggregate effect of solar cost reduction policies on global climate change is ambiguous. But the result improves comparing to the case of competitive market. At least with the existence of market power in the sector of fossil fuels, solar cost reduction policies do not necessarily lead to "Green Paradox". Now we go on to study the impact of cost reduction policies for high cost biofuels. By the proof in the Appendix C, we can have

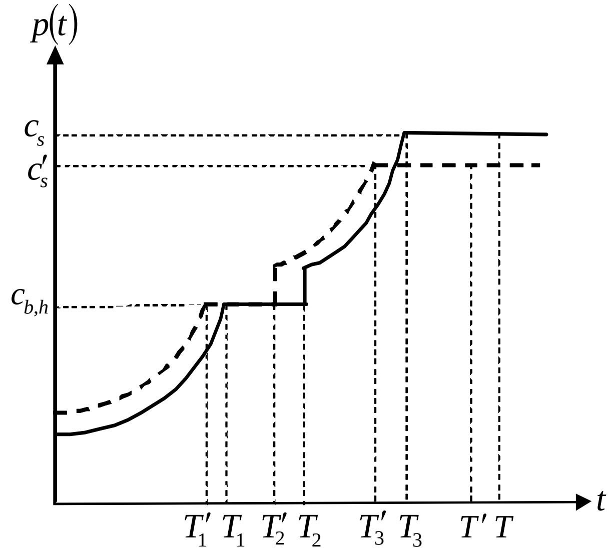

FIGURE 2.11. Solar cost reduction policies: impact on price

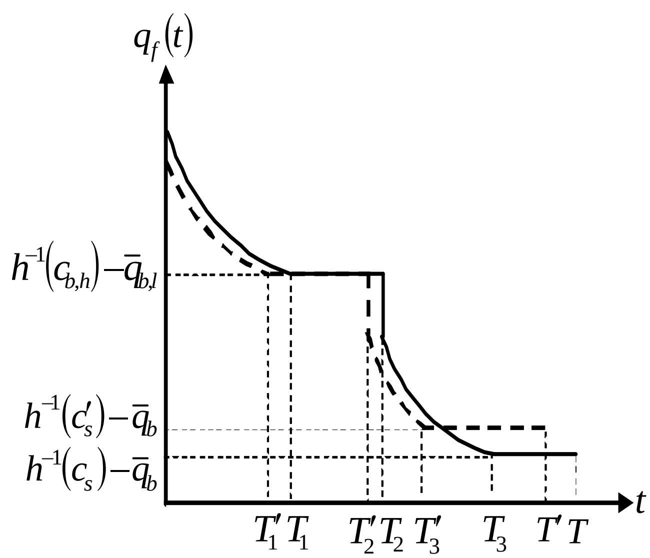

FIGURE 2.12. Solar cost reduction policies: impact on fossil fuel use

Proposition 7. *Under monopoly, cost reduction policies for high cost biofuels*

(1) increase the present shadow value of fossil fuels, i.e. $\partial\mu/\partial c_{b,h} < 0$*;*

(2) bring forward $\{T_1, T_2, T_3, T\}$*, i.e.* $\partial T_i/\partial c_{b,h} > 0$*,* $i = \{1, 2, 3\}$*, and* $\partial T/\partial c_{b,h} > 0$*.*

The results of Proposition 7 are the same as Proposition 6. Again as illustrated in Appendix A, cost reduction policies for high cost biofuels do not uniformly depress the monopolist's future marginal revenue and the effect of increased future marginal damage dominates the other. We illustrate the corresponding policy impacts on price path and extraction path of fossil fuels in Figure 2.13 and 2.14. Although there exists some difference comparing to previous case, the main conclusion regarding to climate change remains the same, i.e. cost reduction policies for high cost biofuels also reduce present extraction of fossil fuels and unfortunately bring forward the exhaustion time of fossil fuels. Whether cost reduction policies for high cost biofuels have "Green Paradox" effect cannot be determined yet. But comparing to the case of competitive market where present extraction of fossil fuel is increased and exhaustion time is postponed, we can learn that the existence of a monopolist in the sector of fossil fuel totally reverses the results of cost reduction policies for high cost biofuels.

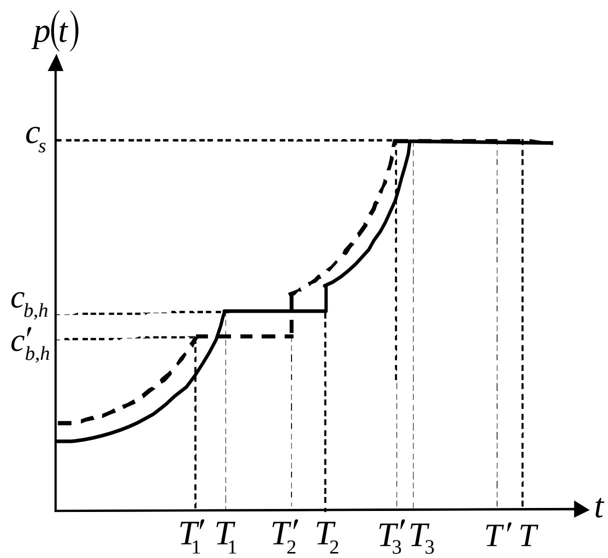

FIGURE 2.13. Cost reduction policies for high cost biofuels: impact on price

Finally we turn to examine the impacts of capacity expansion policies for high cost and low cost biofuels. Unfortunately, the study for these two cases is too complicated

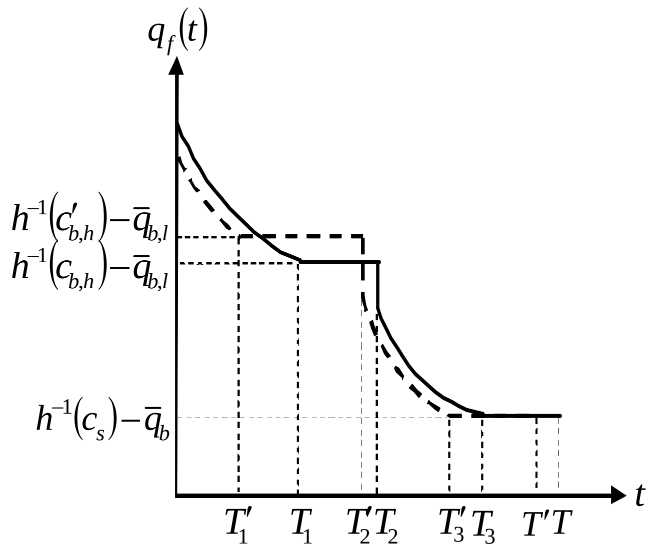

FIGURE 2.14. Cost reduction policies for high cost biofuels: impact on fossil fuel use

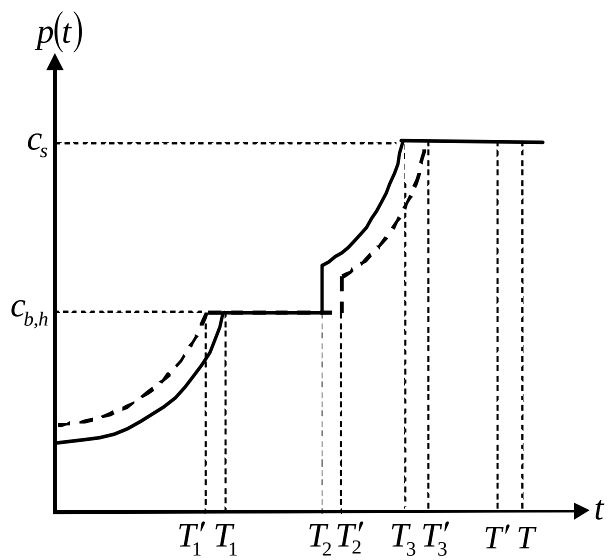

FIGURE 2.15. Capacity expansion policies for high cost biofuels: impact on price

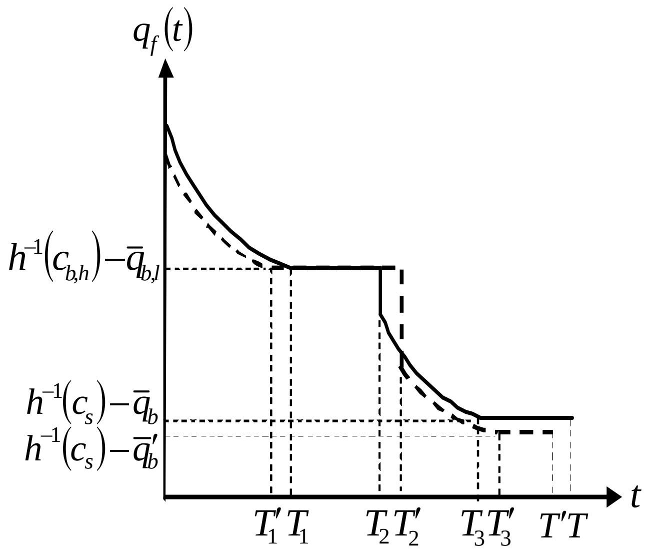

FIGURE 2.16. Capacity expansion policies for high cost biofuels: impact on fossil fuel use

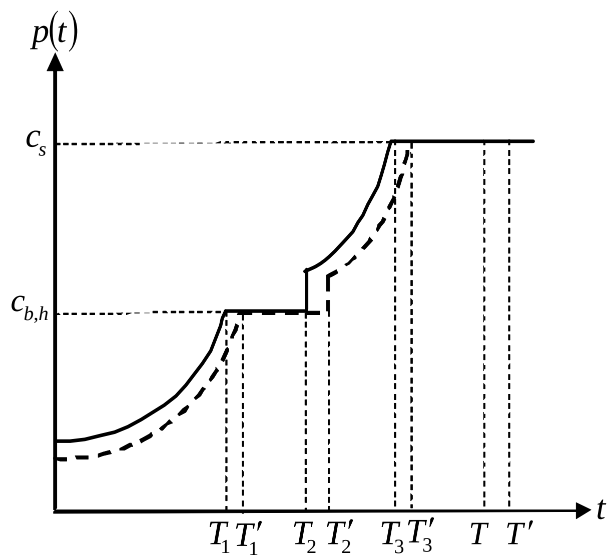

FIGURE 2.17. Capacity expansion policies for low cost biofuels: impact on price

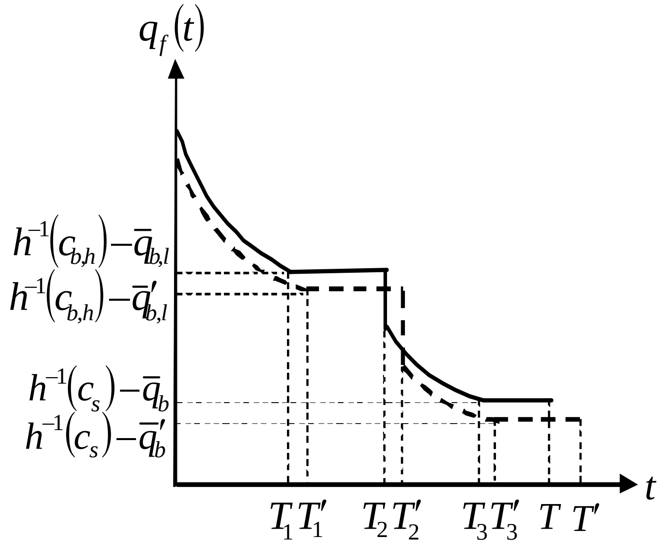

FIGURE 2.18. Capacity expansion policies for low cost biofuels: impact on fossil fuel use

to obtain any analytical result and we have to rely on a numerical example, in which one period lasts for one year, to illustrate the policy impacts. For simplicity, we consider the oil market with a linear demand function, $p_t = \overline{p} - \beta Q_t$. The unit of the production is billion barrels. We roughly set c_f, $c_{b,h}$, c_s and $\overline{p}$ at \$10, \$250, \$380, and \$450. By using the average price, \$73 per barrel (BP, 2010), and average world oil consumption, 31 billion barrels per year (BP, 2010), during $2005 - 2009$, we can compute $\beta = 12$. Initial reserve of fossil fuel is equal to world proved reserves in 2009 which is around 1500 billions (BP, 2010). Let capacity of two biofuels equal to current world production of biofuels, which is around 0.5 billion barrels, we have $\overline{q}_{b,h} = \overline{q}_{b,l} = 0.5$. As standard in the literature, we set interest rate $r = 0.05$. Now by increasing the capacity of biofuels, we can have the corresponding policy impacts as in Table 2.1, where "+" and "-" denote increase and decrease. It needs to be noted that the results in Table 2.1 are robust to any other parameter specification. Based on the numerical example, we illustrate the impacts of those two capacity expansion policies on price path and fossil fuel use in Figure 2.15 ~2.18. Given the results in Table 2.1, the change of fossil fuel use can be easily derived.[9] Same as in the case of competitive market, capacity expansion policies for low cost biofuels satisfy the two standards for environment beneficial policies: reduce

[9]The changes in the second and fourth stage are straightforward. For the first and third stages, we can apply implicit function theorem in (2.13) and (2.16), and follow the same argument in proof for Corollary 2.

TABLE 2.1. Impacts of capacitiy expansion policies in non-competitive market

	μ	T_1	T_2	T_3	T
Low cost biofuels	$-$	$+$	$+$	$+$	$+$
High cost biofuels	$+$	$-$	$+$	$+$	$-$

current supply of fossil fuels and delay the exhaustion time of fossil fuels. On the other hand, capacity expansion policies for high cost biofuels reduce current use of fossil fuels and bring forward the exhaustion time of fossil fuels, which are the same as the impacts of previous two cost reduction policies.

TABLE 2.2. Summary for policy impacts on climage change

	Competitive Market		Non-competitive Market	
	Early use	Exhaustion time	Early use	Exhaustion time
c_s	$-$	$-$	$+$	$-$
$c_{b,h}$	$-$	$+$	$+$	$-$
$\overline{q}_{b,h}$	$-$	$+$	$+$	$-$
$\overline{q}_{b,l}$	$+$	$+$	$+$	$+$

We summarize policy impacts on climate change for all cases in Table 2.2, where "+" and "−" represent positive impact and negative impact on climate change. Table 2.2 shows that "Green Paradox" can only be confirmed in the benchmark case — solar cost reduction policies, and both capacity constraints of renewable energy and existence of market power are possible to lead the "Green Paradox" not to prevail. For the same policy, either direction it can go depending on the capacity constraints and market power. For instance, if the effect of current use of fossil fuel use dominates the exhaustion time of fossil fuels, the existence of market power could turn the negative climate change impacts of all policies to positive. All in all, when we are implementing renewable energy policies to counter global climate change, we should always keep in mind that the response of fossil fuel use to renewable energy policies is more complicated than we can think at the first place and a careful study of the policy impacts is always necessary.

2.5 Conclusion

This paper provides a comprehensive study of impacts of renewable energy policies on energy price, fossil fuel extraction and climate change by emphasizing the roles of capacity constraints of renewable energies and market power. We first examine the benchmark case — solar cost reduction policies in the competitive market, and then extend it by introducing capacity constraints of renewable energies and market power in the sector of fossil fuels.

Our study shows that "Green Paradox" can only be confirmed in the benchmark case and for all other cases, it may not prevail. The main reasons are as follows: the capacity constraints help renewable energy to delay the fossil fuel use to the distant future and the market power in the sector of fossil fuels changes the optimization rule, as well as the response, of fossil fuel owners, in particular it turns the rule from the one depending on market price in competitive market to the one depending on marginal revenue of extracting fossil fuels in non-competitive market. More specifically, if the energy market is competitive, solar cost reduction policies would increase the fossil fuel use in all periods and bring forward the exhaustion time of fossil fuels. But as for high cost biofuel policies, although they increase the fossil fuel use in early stages, they would reduce the fossil fuel use later on after high cost biofuels become competitive and help delay the fossil fuel use to the distant future, postponing the exhaustion time of fossil fuels. For low cost biofuels, since they substitute the use of fossil fuels from the beginning, when their capacity is expanded, they can help reduce early use of fossil fuels as well as extend the exhaustion time of fossil fuels. If we consider the non-competitive market, impacts of capacity expansion policies for low cost biofuels remains the same as those in competitive market, but for all other policies, the results could be reversed. When facing the other three renewable energy policies, the monopolist would respond to increase, instead of reducing, the current price of fossil fuels and thus reduce fossil fuel use at the early stages. As discussed, that would reduce the damage of GHGs.

In this paper, in order to obtain analytical results, we make some simplification. First, we assume marginal cost of producing fossil fuels constant. A more appropriate treatment is that the marginal cost of producing fossil fuels is a function of reserve in the underground $c_f(X_t)$ and is decreasing in it, i.e. $c'_f(X_t) < 0$. If that is the case, solar cost reduction policies have an additional benefit – increasing the unused reserve of fossil fuels in the underground, and our resource exhaustion condition does not hold any more. Specifically, given the existence of abundant renewable energy, a stock of fossil fuels $X = c_f^{-1}(c_s)$ would finally remain unused in the underground. That is because, for any smaller stock of fossil fuels, the extraction cost is higher than solar cost and thus is not economic to supply the market. As c_s falls, unused reserve X increases, reducing carbon emitted to the air. Therefore, if we consider this general cost function, solar cost reduction policies have less chance to generate "Green Paradox" effect. However, if solar cost is high such that $c_s > c_f(0)$, the entire stock of fossil fuels need to be exhausted and our simplification is appropriate. In current technology level, cost of backstop without capacity constraints is pretty high, for example the estimate of backstop cost in Nordhaus (2008) is around \$1000 per ton of carbon. Ploeg and Withagen (2010) have more discussion about this issue.

Second, we assume the marginal cost of renewable energies constant. Given that assumption, the output of renewable energies would jump to full capacity as energy price rises above their production cost. For the reality concern, it would be more appropriate to let the marginal cost of renewable energies increase in the output level until the full capacity. By incorporating that more general cost structure, the quantitative results of our model may change but the qualitative results should still hold.

Third, when we consider non-competitive market, the market structure is simplified. We assume existence of a monopolist controlling the supply of fossil fuels. A more realistic assumption for the sector of fossil fuels is oligopoly. Under that assumption, we need to consider the interactions among oligopoly firms and open-loop or closed-loop dynamic differential game is required to solve the model. This task would be more complicated but by no means trivial, and is left for the future research.

Chapter 3

Health and Precautionary Savings in the Pollution-Growth Nexus

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3.1 Abstract

We study the pollution-economic growth nexus from the perspective of health and precautionary savings. We first establish empirical support from Chinese data that higher pollution levels are associated with high savings rates. We then construct an overlapping-generations model in which agents save more in response to the higher pollution-induced health risk and the increased saving in turn leads to more investment, and thus more pollution. Such a path may appear to be sustainable in terms of economic growth, but the increased pollution makes the welfare level unsustainable. We study three kinds of policy interventions: private insurance achieves full risk sharing but does not reduce pollution; PAYG insurance reduces pollution but can only achieve partial risk sharing; pollution tax reduces pollution, but introduces an additional distortion in the rate of return to capital. A tax on pollution is most effective when the tax revenue is distributed to the old and sick. Even when double dividends do not exist in a static setting, they may still arise in a dynamic setting via its effects on savings behavior.

3.2 Introduction

That a country's environment – the air its citizens breathe, the water they or their livestock drink, and so on – will have a profound effect on its overall performance hardly seems surprising. In fact, it would be almost commonplace to think that poor air or water quality will negatively impact the lives of people and prevent them from being at their productive best. And yet, even as the world economy slowly adjusts to a changing economic order with the BRIC (Brazil, Russia, India and China) countries growing at stupendous rates, images of citizens on the streets of Mumbai or Rio or Beijing, going about their business wearing face masks, are hard to suppress. Could there be a mutually-reinforcing pollution-growth nexus at work? This paper attempts to study such a pollution-growth nexus in market economies, mediated via health concerns.

The literature on this topic seems to have largely ignored the issue of health and its connection with the mutually-reinforcing pollution-growth nexus. In part, this is because existing work tends to focus on the impact of pollution on amenity and productive value, and ask how a more ambitious environmental policy affects long-run growth. In such work, because agents are assumed to have preferences over environmental quality, environmental policy or pollution abatement is a necessity for optimality.[1] Moreover, since pollution abatement always requires a capital input, environmental policy necessarily crowds out capital investment, thereby constraining long-run growth. On the flip side, pollution abatement could lead to an increase in human capital (Gradus and Smulders, 1993) and/or factor productivity (Bovenberg and Smulders, 1995, 1996), and through these channels, enhance long-run growth. It all boils down to which effect of environmental policy dominates the other. Other studies, such as John etc. (1995), Ono (1996), and Stokey (1998) etc., focus on planning solutions for an economy with pollution and study ways to decentralize them via suitable governmental policies.[2] Often, a critical assumption – additively-separable disutility of pollution – is made thereby precluding any direct effect of pollution on the resource allocation problem of the agent. A major theme of our paper is precisely this oft-ignored dimension.

In the last two decades, epidemiological research has found consistent evidence of damaging effects of pollution on human health. A variety of cohort studies (see for example Dockery, 1993 and Pope et al. 1995) and daily time series studies (see for example Samet et al. 2000, Daniels et al. 2000) have respectively shown the long-term and short-term effects of air pollution on morbidity and mortality associated with respiratory and cardiovascular illness.[3] In our model, we incorporate the essence of such damning evidence by assuming that pollution increases the likelihood of poor health in

[1] The literature on environment and growth dates back to Forster (1973) and Gruver (1976) who study how pollution affects intertemporal resource allocation and why optimal capital accumulation should be less than that under the traditional golden rule.

[2] Utilizing the overlapping generations framework, John and Pecchenino (1994) stress the intergenerational conflict of interest. In their study, a one-period lived government collects taxes from the young and use the proceeds to improve old-age environmental quality. They show how different correlations between environmental quality and income are possible, as is the possibility of multiple steady states and overmaintenance of the environment.

[3] Water pollution is another major source of health problems, especially in poor areas that cannot access piped water. WHO (2006) has reported health impacts of every chemical and microbial pollutants in drinking water.

the future.[4]

Specifically consider a standard two-period overlapping-generations model. Agents are assumed to care about their consumption and health in each period. All young agents are fully healthy. An agent may be in poor health status when old, and pollution increases the likelihood of such status. When old, a sick agent can improve his health by incurring medical expenses, and anticipating this possibility, a young agent makes "precautionary savings." Higher pollution levels, by raising the health risk for the old, provide impetus to more precautionary savings. In a closed economy, more saving raises the capital stock in equilibrium. Consequently, higher pollution levels lead to higher capital stock levels, which in turn causes more pollution. In the market economy, pollution and growth tend to reinforce each other along the growth path. Such a path looks attractive from the standpoint of economic growth but the increased pollution seriously hurts welfare.

Because of the public-good nature of pollution, the market solution is not efficient. There are two main inefficiencies: the lack of consumption smoothing or risk sharing for the old, and the absence of mechanisms to internalize the pollution externality of capital. We study policy interventions that address, directly or indirectly, these two inefficiencies. We first evaluate two common stabilization instruments, private health insurance and pay-as-you-go (PAYG) insurance akin to the U.S. Medicare system. We then evaluate the effects of a Pigouvian tax as well as a set of first best policies. We find that private insurance achieves full risk sharing but does not reduce pollution. While PAYG insurance reduces pollution, it can only achieve partial risk sharing. A pollution tax reduces pollution, but introduces an additional distortion in the rate of return to capital.

This paper also adds to the literature explaining the coexistence of high savings rates and high pollution in many developing countries.[5] We collect a cross section of air-pollution and savings data from urban China for 2002 and test the effect of pollution on savings rates. Controlling for the endogeneity of air pollution, the empirical evidence

[4] Of course, the negative effect on health is certainly not the only damage that pollution generates. But it is undoubtedly the costliest among all damages, based on contingent valuation studies of willingness to pay for pollution reductions. For example, in 2003 the estimated mean monetary health costs of air and water pollution in China were 520 billion yuan (3.8 percent of GDP) and 66 billion yuan respectively. In contrast, the non-health damage which includes crop loss, fishery loss and material erosion etc., was about 50 billion yuan (World Bank, 2007). Similar evidence can be found in developed countries. The U.S. EPA (1997) estimates that during 1970-1990, the mean value of total monetized human health benefits of the Clean Air Act was $22 trillion. In contrast, the non-health gains in household cleanliness, agriculture and visibility value were $74 billion, $11 billion, and $38 billion respectively.

[5] The high savings rate in China is generally explained by life-cycle theory (Modigliani and Cao, 2004) with financial underdevelopment and precautionary savings caused by market reforms (Chamon and Prasad, 2007). In a recent paper, Wei and Zhang (2008) show that the one child policy lead to an imbalance of the sex ratio, which also can help explain the high savings rate in China.

confirms a positive and significant effect of pollution on the savings rate, motivating our theoretical investigation.

Our paper belongs to a short line of papers investigating the nexus between pollution, health, and economic activity. William (2002, 2003) introduces health in a static model to re-examine the "double dividend" hypothesis of environmental taxes, where pollution increases sickness time and medical expenses. In these models, a Pigouvian tax typically causes welfare losses, because the health effects aggravate distortions in labor market. In their dynamic models, Jouvet et al. (2007) and Pautrel (2008) assume that life expectancy is negatively influenced by pollution. Pautrel (2008) examines the long-run growth effects of pollution by assuming that agents inherit human capital from past generations and shortened life expectancy constrains the accumulation of human capital.[6] Gutierrez (2008) studies a model similar to ours in which agents need to save more when pollution is high. It is noteworthy that Gutierrez (2008) does not explicitly model the effect of pollution on health nor deal with the issue of health risk and health capital; in fact, pollution-determined health expenditure is treated as exogenous to an individual.

The rest of the paper is organized as follows. Section 3.3 presents empirical evidence showing increased savings in response to higher pollution. Section 3.4 analyzes the effects of pollution on capital accumulation in an overlapping generations economy without health insurance or government intervention. We then solve the social planner's problem in section 3.5. We next (in Section 3.6) compare different health insurance systems and environmental policies, and design the optimal policy scheme. We conclude in Section 3.7. Appendix B describes the data used, and Proofs of major results are in Appendix **??**.

3.3 Pollution and Savings: Empirical Evidence

In our model, agents save more because they undertake precautionary savings in response to the higher health risks due to pollution. We first present empirical evidence showing that household savings increase in response to higher pollution levels. There is already a large empirical literature documenting the negative impact of pollution on health. Our empirical model investigates a related but different subject: whether pollution affects the savings rate using the Chinese data. China in recent years has experienced rapid

[6] One can quibble with the idea that the most important and direct effect of pollution is on life expectancy. If the connection between these two were that strong, one would expect high pollution to discourage saving as lifespans become shorter. Interestingly, our own empirical findings do not support such a claim. There is also the issue that many low-pollution countries are also those with low life expectancy.

economic growth as well as increased pollution levels. There is also widespread cross sectional variation in growth rates and pollution levels within China.

The primary data source is the Chinese national urban household survey data in 2002. Because drinking water systems are well developed in urban areas of China, we consider air pollution at the city level. Appendix B describes the data sources and summary statistics of the variables used. The econometric model is

$$H\_Save_{ij} = \beta_0 + X_{ij}\boldsymbol{\beta} + C_j\boldsymbol{\gamma} + \epsilon_{ij},$$

where i indexes the households (4777 in total) and j indexes cities (39 in total). H_Save is the household savings rate. Household variables X_{ij} include income, household size, and other important social economic characteristics of the household and the household head. City variables C_j include, among others, the level of air pollution. Estimation results are presented in Table 3.1.

We first run Ordinary Least Squares (OLS) regression and the results are reported in column (1) of Table 1. The household characteristics have expected signs. For instance, a household saves more as its income increases (but at a decreasing rate), as the household is larger, and/or as it has more children (i.e., saving for future expenses). The household saves less as more family members are currently ill, since the disposable income is lower. The household saves more as the household head is older, but saves less as the head is more educated (possibly because the household head has a more stable job).

Air pollution (C_Air) significantly raises the household savings rate. However, pollution is partly generated by production activities that might be affected by savings, i.e., variable C_Air might be correlated with the error term and might be endogenous – this is confirmed by the Durbin-Wu-Hausman test reported in Table 1. To find appropriate instruments for C_Air, we exploit exogenous variations in air pollution due to varying rainfall levels and wind speeds across cities. Rainfall washes away pollutants from the air and high wind blows air pollutants away from cities. Both variables are unlikely to be correlated with unobserved determinants of household savings. We thus run 2SLS regressions in which wind (C_Wind) and rainfall (C_Rain) are used as instruments for C_Air.

Column (2) of Table 1 reports the first stage regression results. Both C_Wind and C_Rain are significant at 1% significance level and have the expected signs. The coefficients imply that an increase by 1000 milliliters of rainfall and by 10 meters/second of wind speed could respectively lower the air pollution index by 0.87 and 0.3.

TABLE 3.1. Ordinary Least Squares (OLS) Regression and Two-stage Least Squares (2SLS) Regressions of Household Savings Rate on Air Pollution

	OLS	1st Stage of 2SLS	2SLS
	H_Save	C_Air	H_Save
	(1)	(2)	(3)
C_Air	4.036***		2.494*
	(1.209)		(1.304)
H_Inc	3.370***	0.001	3.372***
	(0.262)	(0.005)	(0.256)
H_Inc2	-0.049***	0.0002	-0.049***
	(0.006)	(0.0001)	(0.006)
H_Size	2.023***	0.025	2.036***
	(0.525)	(0.017)	(0.514)
H_Emp	0.665	-0.054***	0.569
	(0.614)	(0.017)	(0.633)
H_Child	1.767*	-0.030	1.812*
	(0.973)	(0.0307)	(0.956)
H_Hcap	-0.449	-0.051***	-0.568
	(0.421)	(0.014)	(0.391)
H_Ill	-2.839***	-0.020	-2.853***
	(0.706)	(0.018)	(0.68)
H_Insur	0.065	-0.012	-0.018
	(0.309)	(0.012)	(0.313)
HH_Age	0.164***	-0.003*	0.165***
	(0.047)	(0.001)	(0.047)
HH_Edu	-0.318**	-0.0006	-0.317***
	(0.125)	(0.004)	(0.124)
HH_Sex	1.362**	0.072***	1.611**
	(0.644)	(0.022)	(0.687)

Table continued

	OLS	1st Stage of 2SLS	2SLS
	H_Save	C_Air	H_Save
	(1)	(2)	(3)
C_Pop	-0.031***	0.002***	-0.026***
	(0.004)	(0.00004)	(0.005)
C_Manu	-0.101**	0.046***	-0.037
	(0.066)	(0.001)	(0.084)
C_Inc	-0.089	-0.154***	-0.566
	(0.532)	(0.010)	(0.687)
C_Wind		-0.302***	
		(0.017)	
C_Rain		-0.873***	
		(0.028)	
R^2	0.200	0.518	0.197
N	4777	4777	4777

Note: Symbols *, ** and *** denote significant level at 10%, 5% and 1%. Cluster (by city) robust standard errors are in parentheses. The two instruments pass Wooldridge's robust score over-identification test with the χ^2 statistic equal to 0.812, failing to reject the null hypothesis of their validity. The Durbin-Wu-Hausman chi-sq test for the 2SLS regression is 7.151, rejecting the null hypothesis of exogeneity of C_Air.

Column (3) of Table 1 reports results of the second stage regression. The Durbin-Wu-Hausman chi-sq test confirms the endogeneity of C_Air, and the two instrumental variables pass the over-identification test of exogeneity. Hence, the 2SLS results provide consistent estimates of the impacts of air pollution on savings. The coefficient of air pollution is positive but is now significant only at 10% level (it was significant at 1% level in Column (1)). Compared with the OLS estimates, the coefficient of C_Air in 2SLS is also lower, which is consistent with expectations. The 2SLS estimates imply that, as city air pollution index increases by 1 unit, the household savings rate goes up by 2.5 percentage points. Overall, the estimation results provide strong evidence that

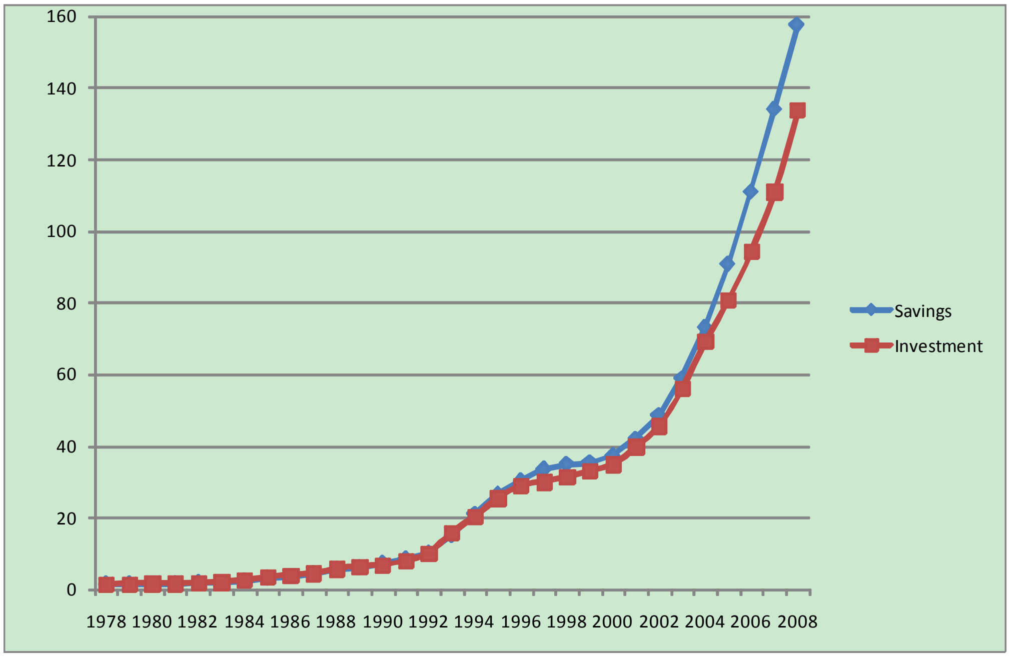


FIGURE 3.1. The Relationship Between National Savings and Total Investment in China

households save more in response to higher pollution levels.[7]

Based on the empirical evidence, we construct a *closed economy* model in the next section describing how pollution leads to increased savings. At least for developing countries, a closed economy provides a good approximation for our purpose since these countries do not invest heavily in foreign nations. Figure 3.1 shows that in China, the aggregate investment closely tracks the total national savings.

3.4 Pollution and Savings: A Theoretical Model

We posit a mechanism through which increased pollution leads to higher savings: increased pollution causes higher likelihood of getting sick (as confirmed by the empirical literature linking pollution to health), and anticipating this, agents respond by undertaking more precautionary savings.[8] We study an economy consisting of an infinite sequence of two-period lived overlapping generations, an initial-old generation, and an infinitely-lived government. Let $t = 1, 2, ...$ index time. At each date t, a new generation is born,

[7] Moreover, our empirical results provide a new angle to explain the high savings rate in China, which results in a large current account surplus and rising foreign exchange reserves.

[8] We do not require the agents to be fully aware of the linkage between pollution and health risks; we only require them to be responsive to increased health risks.

which is comprised of a continuum of identical members assumed to be of measure one. Each agent is endowed with one unit of labor when young and is retired when old. Additionally, a young agent is endowed with one unit of health capital. As will be shortly evident, each young agent is healthy but an old agent is not necessarily so.

There is a single final good produced using a constant returns to scale production function $F(K_t, L_t)$ where K_t denotes the capital input and L_t denotes the labor input at t. Since $L_t = 1$ we know the capital-labor ratio (capital per young agent) is $k_t \equiv K_t/L_t = K_t$. Then, output per young agent at time t can be expressed as $f(k_t)$ where $f(k_t) \equiv F(K_t, 1)$ is the intensive production function. We assume that f takes the Cobb-Douglas form, i,e.,

$$f(k_t) = Ak_t^\alpha, \ A > 0, \ \alpha \in (0, 1). \tag{3.1}$$

The final good can either be consumed in the period it is produced, or it can be saved to provide capital the following period. Capital is assumed to depreciate 100% between periods.[9]

Young agents supply labor inelastically in competitive labor markets, earning a wage of w_t at time t, where

$$w_t \equiv w(k_t) = f(k_t) - k_t f'(k_t) = (1 - \alpha) Ak_t^\alpha. \tag{3.2}$$

Capital is traded in competitive capital markets, and earns a gross real return of R_{t+1} between t and $t + 1$, where

$$R_{t+1} \equiv R(k_{t+1}) = f'(k_{t+1}) = \alpha Ak_{t+1}^{\alpha-1}, \tag{3.3}$$

with $R'(k_{t+1}) < 0$.

As is fairly standard, pollution is modeled as an "inevitable side-product" of production related activity by firms. For analytical tractability, it is assumed that pollution is a by-product of capital use itself. As will become clear, pollution is a public bad – it hurts the health status of agents. It can be mitigated by reducing capital use or by active, costly abatement. For generality, we model stock pollutants: pollution P_t dissipates gradually:

$$P_{t+1} = (1 - \zeta)P_t + \rho k_t - G(q_t) \tag{3.4}$$

[9]This assumption greatly simplifies our analysis without affecting the major results.

where ζ is the natural absorption coefficient, q_t is the total expenditure on pollution abatement and $G(q_t)$ describes the abatement technology. We assume $G(q_t)$ non-decreasing and concave in q_t with $G(0) = 0$ and $\lim_{q_t \to 0} G'(q_t) = \infty$. Coefficient ρ denotes the amount of pollution generated by one unit of capital use. Nations differ in this pollution intensity due to heterogeneity in their endowments, technologies, and industrial structures. We will study the long-run growth and health implications of such differences. Since pollution is a public bad, neither consumers nor producers (there are infinite numbers of both) have any incentive to pay for the abatement activity; hence $q_t = 0$ without government intervention.

While all young agents are assumed healthy, the health status of the old is random and can be either good or poor. We assume that the health status is realized at the start of the second period of life. An old agent (born at date t) finds himself in poor health at the start of date $t+1$ with probability σ where σ depends on the existing, aggregate pollution level P_{t+1}:

$$\sigma_{t+1} = \sigma(P_{t+1}), \tag{3.5}$$

and $\sigma(\cdot)$ is a non-decreasing, concave function of P satisfying $\sigma(\cdot) \in [0,1)$. Parameter $\sigma(0) > 0$ is an exogenous summary measure of the basic health level of the economy.[10] Private agents take σ parametrically when solving their own problems.

If an old agent is in good health at $t+1$, his health capital is denoted as h^o_{t+1} and is normalized at $h^o_{t+1} = 1$ (i.e., the agent maintains his health status from youth). If the agent is in poor health, his health capital is denoted as $h^{o,d}_{t+1}$, and in this case the agent can influence $h^{o,d}_{t+1}$ by incurring medical expenditures m:

$$h^{o,d}_{t+1} = \psi m^{\psi}_{t+1}, \tag{3.6}$$

where $\psi \in (0,1)$ is a parameter. To guarantee that $h^{o,d}_{t+1} < 1 = h^o_{t+1}$, i.e., medical expenses can never restore the full health status of youth to the sick, we assume

Assumption 1 *Parameter $\psi \in (0,1)$ is sufficiently small so that* [11]

$$\psi m^{\psi}_{t+1} < 1 \quad \forall t.$$

[10]Presumably, poorer countries with small, inefficient public health systems, have higher levels of $\sigma(0)$.

[11]As we will show below, the pollution level P_t and medical expenditure m_t are increasing along the optimal path, and eventually m_t increases to the steady state level $\overline{m}$. Then a sufficient condition for this inequality is $\psi\overline{m}^{\psi} < 1$.

Let δ be the subjective discount rate, c_t^y be the consumption of the young born at date t, $c_{t+1}^{o,d}$ be the consumption of the old agent at date $t+1$ who is in poor health, and c_{t+1}^o be the consumption of the old agent at date $t+1$ who is in good health. We posit that agents care both about consumption and their health in old age. Specifically, the expected life time utility of an agent born at date t is

$$U_t \equiv \ln c_t^y + \delta \left\{ \sigma_{t+1} \left[\omega \ln h_{t+1}^{o,d} + \ln c_{t+1}^{o,d} \right] + (1 - \sigma_{t+1})] \ln c_{t+1}^o \right\}, \quad t = 1, 2, \ldots \tag{3.7}$$

where $\omega \in (0,1)$ is the utility weight on health. Note that (3.7) incorporates the payoff from good health: $\omega \ln h_{t+1}^o = 0$ since $h_{t+1}^o = 1$ for the old and $w \ln 1 = 0$ for the young.

The agent faces the following per-period budget constraints:

$$s_t + c_t^y = w_t \tag{3.8}$$

$$c_{t+1}^{o,d} + m_{t+1} = s_t R_{t+1} \tag{3.9}$$

$$c_{t+1}^o = s_t R_{t+1}. \tag{3.10}$$

In (3.8), the agent when young allocates his labor income between consumption and saving. Equation (3.9) is the budget constraint for the old in poor health: he allocates his income between consumption and medical expenses. Equation (3.10) is the budget constraint for the old in good health. In equilibrium, the gross rate of return R_{t+1} is given in (3.3) with $k_{t+1} = s_t$ (since capital is assumed to depreciate completely after a period).

3.4.1 Market Equilibrium

With perfect foresight regarding R_{t+1}, and taking w_t in (3.2) and $\sigma(\cdot)$ in (3.5) as given, a young agent at date t chooses c_t^y, s_t, m_{t+1}, $c_{t+1}^{o,d}$ and c_{t+1}^o to maximize the expected lifetime utility (3.7), subject to (3.8) - (3.10) as well as the health capital production function (3.6). It is easy to check that old agents in poor health spend a constant fraction $\omega\psi/(1+\omega\psi)$ of their income towards medical expenses:

$$m_{t+1} = \left(\frac{\omega\psi}{1+\omega\psi} \right) s_t R_{t+1}. \tag{3.11}$$

Using (3.11), the optimal savings of the young take the form

$$s_t = \theta_t w_t, \quad \text{with} \quad \theta_t = \theta(P_{t+1}) \equiv \frac{\omega\psi\delta\sigma(P_{t+1}) + \delta}{\omega\psi\delta\sigma(P_{t+1}) + \delta + 1}, \tag{3.12}$$

where, θ_t, the savings rate, is increasing in the probability of poor health status $\sigma(\cdot)$. This is the precautionary saving motive at play. If poor health risk rises, future consumption becomes more risky, and agents, in an attempt to smooth the risk, save more. Equation (3.12) also indicates that

Lemma 8. *Under the log-utility assumption, the savings rate and thus consumption c_t^y are independent of the capital rate of return R_{t+1}.*

Substituting equilibrium prices and the general-equilibrium condition $s_t = k_{t+1}$ into (3.12) and (3.4), and using (3.2) and the fact that $q_t = 0$ without government intervention, we obtain a two-dimensional dynamic system of the economy:

$$k_{t+1} = \theta\left(P_{t+1}\right) \cdot (1-\alpha) A k_t^{\alpha} \tag{3.13}$$

$$P_{t+1} = (1-\zeta) P_t + \rho k_t. \tag{3.14}$$

Given k_0 and P_0, the dynamic competitive equilibria are characterized by sequences of $\left\{k_t, m_t, P_t, c_t^y, c_t^{o,d}, c_t^o\right\}$ that satisfy equations (3.13) and (3.14) where $\theta(\cdot)$ is defined in (3.12).

It is instructive to isolate the main channels of action in this economy. Along a growth path in which capital use is increasing, the stock of pollution is rising (since pollution is an inevitable by-product of capital use). Pollution makes it more likely that old agents have poor health. In that event, agents would substitute out of consumption into medical expenses. This heightened consumption risk raises the precautionary motive for saving, causing the equilibrium capital stock to rise and feed into pollution. Thus, pollution and capital accumulation tend to reinforce each other, by way of and at the cost of worsening health. To see this more clearly, consider two economies identical in all respects, except for ρ, the pollution intensities of their GDP. It follows from our discussion above that the per capita income of the high-ρ (i.e., more pollution intensive) economy will always be above the low-ρ economy. However, as shown in the following section, the more pollution intensive economy will generally have lower aggregate welfare.

3.4.2 Stationary Equilibria

At the steady state, $k_{t+1} = k_t = \overline{k}$ and $P_{t+1} = P_t = \overline{P}$. Using (3.13) and (3.14), $\left(\overline{k}, \overline{P}\right)$ are jointly determined by

$$\overline{k} = \frac{\omega\psi\delta\sigma\left(\overline{P}\right) + \delta}{\omega\psi\delta\sigma\left(\overline{P}\right) + \delta + 1}(1-\alpha)A\overline{k}^{\alpha} = \overline{\theta}(1-\alpha)A\overline{k}^{\alpha} \tag{3.15}$$

$$\overline{P} = \frac{\rho\overline{k}}{\zeta}, \tag{3.16}$$

where $\overline{\theta} = \theta(\overline{P})$. The following result shows the existence, uniqueness, and stability properties of the steady-state equilibrium.

Proposition 9. *(i)* $\left(\overline{k}, \overline{P}\right) = (0, 0)$ *is a steady state equilibrium and is locally unstable. (ii) There exists a locally-stable, non-trivial steady state. (iii) A sufficient condition for the uniqueness of the steady state is* $\left[2 + \delta + (\delta + 1) / (\omega\psi)\right](1-\alpha) > 1$.

The proof is in Appendix **??**. The sufficient condition in (iii) is easily satisfied by typical economies. For instance, $\alpha \leq 0.5$ in a typical economy, which guarantees this sufficient condition. Even when $\alpha > 0.5$, the condition is satisfied when the discount factor δ is not too low, or when the health parameter ψ is low (consistent with Assumption 3.4). Henceforth, we assume that the parameter configuration is such that the non-trivial steady state is unique. The phase-diagram is illustrated in Figure 3.2.

We next show how pollution affects the steady state.

Proposition 10. *Given that there is a unique non-trivial steady state, then*

$$\frac{\partial\overline{k}}{\partial\rho} > 0 \quad \textit{and} \quad \frac{\partial\overline{k}}{\partial\zeta} < 0.$$

When capital becomes more pollution-intensive (higher ρ), or when pollution dissipates more slowly (lower ξ), both the steady state capital stock $\bar{k}$ and pollution level $\bar{P}$ increase. Income of both the old and the young rises but the health risk or population of old in poor health also rises.

Therefore, an increase in pollution intensity of capital ρ has two opposing effects on long-run social welfare, i.e., the utility level achieved in the steady state: the capital effect and the health effect. The net effect on welfare is ambiguous in some cases, as the next Proposition shows.

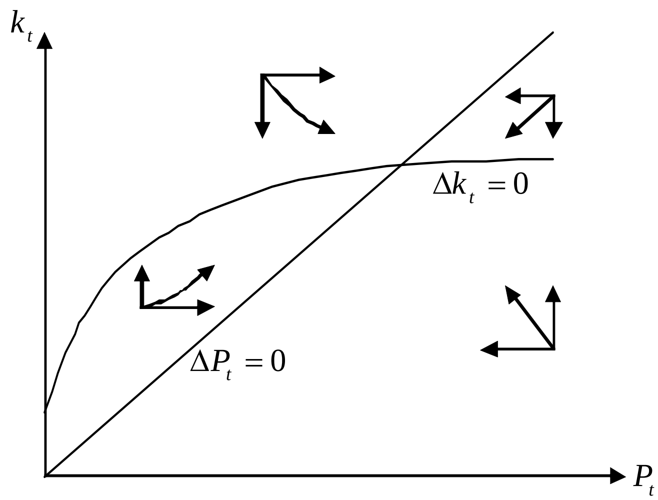


FIGURE 3.2. Dynamics of pollution and capital with unique steady state

Proposition 11. *Let $\widehat{k}$ be such that $f'(\widehat{k}) \equiv 1$: $\widehat{k}$ is the golden rule capital level without pollution (when $\rho = 0$).*
(i) If the steady state capital level $\overline{k} \geq \widehat{k}$, the steady state utility is monotonically deceasing in ρ.
(ii) If $\overline{k} < \widehat{k}$, the sign of $\partial U/\partial \rho$ is ambiguous.

Since the aforediscussed health effect always reduces welfare, the impact of higher ρ (pollution) on social welfare depends on whether or not over-accumulation of capital is beneficial. Without pollution, the golden rule capital level $\widehat{k}$ maximizes steady state utility, but with pollution and the associated health damages, as will be shown in section 3.5, the optimal capital level (a "modified" golden rule level) is lower than $\widehat{k}$. When $\overline{k} \geq \widehat{k}$, even without taking account of its externality, further increase of $\overline{k}$ always reduces welfare. In this case, the capital effect and the health effect work in the same direction, and both imply that the long-run social welfare is monotonically decreasing in ρ. In contrast, if $\overline{k} < \widehat{k}$, more accumulation of capital might be beneficial for the long-run welfare: in this case, the capital effect and the health effect have opposite welfare impacts, and the capital effect might dominate the health effect, so that steady state utility increases in ρ. This is confirmed by the numerical example in Section 3.4.3. The literature has extensively discussed over-accumulation of capital in OLG models. There are also many reasons why an economy might under-accumulate capital, e.g., when individuals have high discount rates. When this happens, polluting capital and precautionary saving might in fact improve the steady state utility level. For these economies, a higher ρ (more pollution intensive technologies) can increase both income and welfare.

3.4.3 An example

Since the effect of a more-polluting technology choice (higher ρ) is ambiguous for $\overline{k} < \widehat{k}$, we proceed to study this issue via a numerical example. The numerical example is roughly calibrated to after-reform China.[12]

In this example, each period lasts for 25 years, i.e., the remaining life expectancy of an agent entering the workforce is 50 years. Scale parameter A is set at 3. Following Young (2003), we set $\alpha = 0.4$. For this parametric specification, $\widehat{k} = 1.36$. We set $\delta = 0.99^{25} = 0.78$.

From (3.12), (3.2) and (3.1), we know the steady state savings rate is given by

$$\frac{\overline{s}}{f(\overline{k})} = \frac{\omega\psi\delta\sigma\left(\overline{P}\right) + \delta}{\omega\psi\delta\sigma\left(\overline{P}\right) + \delta + 1}(1 - \alpha).$$

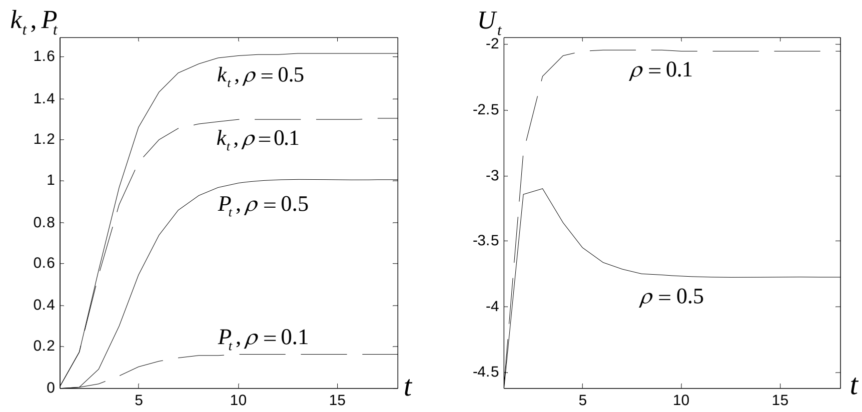
From China Statistical Yearbook (2008), the average national savings rate in China is about 0.39 over the period 1978-2008. Setting $\overline{s}/f(\overline{k}) = 0.39$, we obtain $\sigma(\overline{P})\omega\psi = 1.38$. From the China Health Statistics Yearbooks, we compute $\sigma(\overline{P}) = 0.38$,[13] implying that $\omega\psi = 3.63$. We set $\omega = 7.26$ and $\psi = 0.5$. Since the utility from health is in logarithmic form and is additively separable from that of consumption, the model results depend only on the product of $\omega\psi$, instead of individual values of ω and ψ (as long as ψ is sufficiently low, see Assumption 3.4). For flow pollutants (e.g., PM_{10} or some water pollutants), the dissipation parameter $\zeta = 1$, but ζ can be low for stock pollutants (e.g., lead and CO_2). We set $\zeta = 0.8$.

Finally we assume σ follows exponential distribution, $\sigma\left(P_t\right) = 1 - \exp\left(-0.32 - P_t\right)$, such that pollution-induced chronic illness among the elderly accounts for about 28 percentage of all chronic illnesses (i.e., $\sigma(0)/\sigma(\overline{P}) = 0.72$).

Since pollution and capital accumulation are jointly and endogenously determined, it is not possible to ask questions of the form: how does capital accumulation or welfare

[12] We use the Chinese example to be consistent with the empirical section (Section 3.3), which is based on the Chinese data.

[13] The Chinese Ministry of Health conducted three national health surveys in 1993, 1998 and 2003, and *China Health Statistics Yearbook 2004* publishes the morbidity rate of chronic diseases in different age groups. With corresponding population data from *China Population Statistics Yearbook*, we calculate three morbidity rate of chronic diseases of people above age 40 in 1993 and above age 45 in 1998 and 2003 (the cut-off ages changed from 40 to 45 in the 1998 and 2003 surveys), and the average is 0.375.

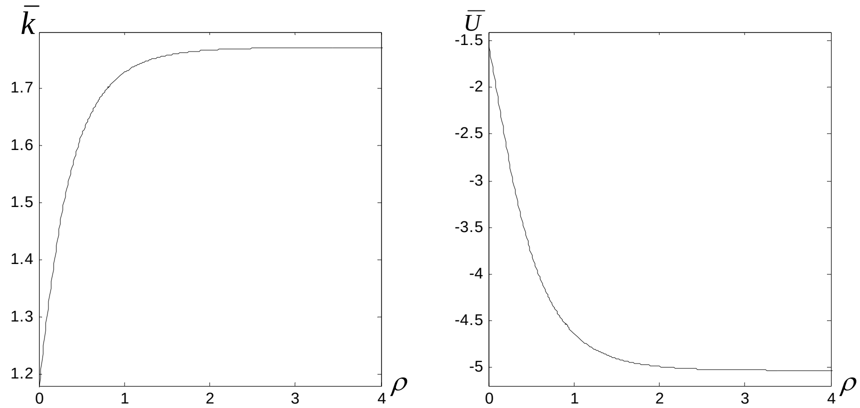


(a) Path of capital and pollution (b) Path of utility

FIGURE 3.3. Capital, pollution and utility paths

respond to increased pollution. To get at a similar idea, we instead study the effects of pollution intensity ρ (via pollution-technology choice) on capital and social welfare. Setting $k_0 = 0.001$ and $P_0 = 0$, we obtain different growth paths of capital, pollution and welfare for $\rho = 0.1$ and $\rho = 0.5$ in Figure 3.3. Panel (a) is consistent with Proposition 10: a higher ρ always implies higher capital and higher pollution levels. However, Figure 3.3(b) shows that higher ρ reduces agents' lifetime utilities. Moreover, when $\rho = 0.1$, individual utilities monotonically increase over time, i.e., each generation's utility is higher than those of the previous generations, following a similar pattern to capital k_t and pollution P_t. But when $\rho = 0.5$, the utilities rise for the first three periods and then starts to fall, although both k_t and P_t are increasing all the time. *Although the economy seems to be on a "sustainable path" with increasing capital and GDP, the accompanying increase in pollution makes the welfare achieved in early periods unsustainable.* Intuitively, when both capital and pollution are low, the capital effect of pollution dominates the health effect of pollution and thus the utility is increasing. But eventually the pollution effect dominates, leading U_t to decrease as capital and pollution further rise.

Figure 3.4 shows the effects of pollution intensity ρ on the steady state capital $\overline{k}$ and utility $\overline{U}$, when $\overline{k}$ is sufficiently high. Consistent with Proposition 11, although $\overline{k}$ is monotonically increasing in ρ, $\overline{U}$ is monotonically decreasing in ρ both when $\overline{k} > \widehat{k} = 1.36$ (Proposition 11(i)) and when $\overline{k} < \widehat{k}$ but $\overline{k}$ is not too small (Proposition 11(iii)). For $\overline{k} \in [1.2, 1.7]$, the health effect significantly dominates the capital effect: the social welfare decreases drastically as ρ rises.



(a) Steady state capital (b) Steady state utility

FIGURE 3.4. Effects of ρ on steady state capital and utility

On the other hand, when $\overline{k}$ is sufficiently low, increased pollution intensity *can* improve steady state utilities. To illustrate this possibility, we set $\delta = 0.08$[14] and show the effects of higher ρ values in Figure 3.5. When $\overline{k} < 0.0165$, the capital effect dominates: a higher ρ raises the steady state utility levels. But after $\overline{k}$ reaches a threshold value, the steady state welfare decreases. In an economy with extremely low capital, pollution might help raise the savings rate, improving welfare in the long run.

3.5 Social Optimum

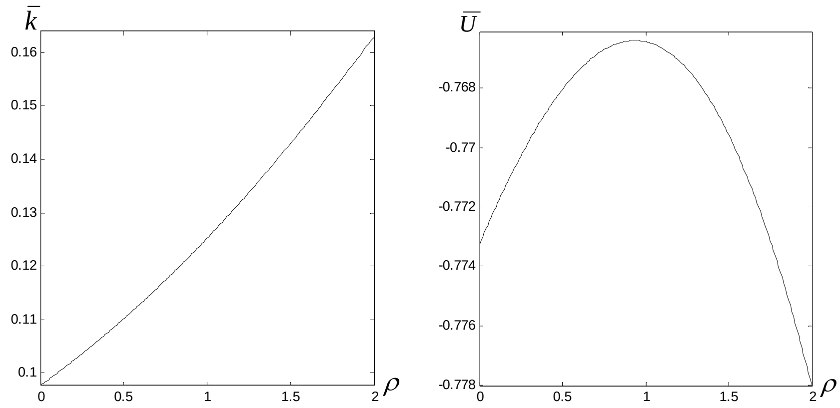
Clearly, because of the public good nature of pollution, the market solution is not efficient. In this section, we characterize the social optimum, the optimal path chosen by a social planner. Unlike the market, the planner will allocate goods towards pollution abatement. Let q_t denote the pollution-abatement expenditure per worker. Then the social planner's problem is to maximize the expected lifetime utility of all current and future generations:

$$\max_{\left\{k_t, q_t, P_t, m_t, c_t^y, c_t^{o,d}, c_t^o\right\}} \sum_{t=0}^{\infty} \beta^t U_t, \tag{3.17}$$

subject to the pollution-generation function (3.4), the resource constraint

$$c_t^y + \sigma\left(P_t\right)\left(c_t^{o,d} + m_t\right) + \left[1 - \sigma\left(P_t\right)\right] c_t^o + k_{t+1} + q_t = f\left(k_t\right), \quad t = 1, 2, \ldots \tag{3.18}$$

[14]This is the discount factor for 25 years, and is equivalent to a discount factor of 0.904 or a discount rate of 10% per year.



(a) Steady state capital (b) Steady state utility

FIGURE 3.5. Pollution improving steady state utility

$q_t \geq 0$, $P_t \geq 0$, and the initial capital k_0 and pollution P_0. In (3.17), β is the subjective discount factor of the planner, which might be different from δ, an agent's discount factor. The agent's utility U_t is given in (3.7), and $U_0 = \sigma(P_1)\left[\omega \ln h_1^{o,d} + \ln c_1^{o,d}\right] + (1-\sigma(P_1)) \ln c_1^o$ is the expected utility of the initial old population.[15] In (3.18), the planner allocates output towards young-age consumption, the expected consumption and medical expenses of the old, investment, and pollution abatement. Unlike private agents, the planner understands the effect of pollution on the probability of being in good health.

Let

$$\begin{aligned}\Omega_t &= \sigma'(a + P_t)\left[\beta m_t \beta \lambda_{t+1} - \delta\omega \ln\left(\psi m_t^{\psi}\right)\right] \\ &= \delta\omega\sigma'(a + P_t)\left[\psi - \ln\left(\psi m_t^{\psi}\right)\right] > 0\end{aligned} \tag{3.19}$$

be the marginal damage from pollution P_t in period t, and

$$D_t = \sum_{i=t}^{\infty} [\beta(1-\zeta)]^{i-t}\, \Omega_{i+1} \tag{3.20}$$

be the present value of the marginal damage of pollution P_t in all periods. In (3.19), pollution P_t, through increasing health risk (by $\sigma'(P_t)$), imposes two costs on the society in period t: (i) each sick old agent incurs health expenditure m_t, reducing the capital available for period $t+1$ (cf. (3.18)). Capital k_{t+1} is valued at $\beta\lambda_{t+1}$ in period $t+1$ (where

[15] An alternative formulation of the planner's problem is to maximize an agent's steady state utility level $\overline{U}$ (cf. (3.7)). This is equivalent to a special case of (3.17) with $\beta = 1$ and the results of this section (modified by $\beta = 1$) still apply.

$\beta\lambda_{t+1}$ is the Lagrangian multiplier of (3.18)). Discounting to period t at rate β, the social cost of this health expenditure is $\beta m_t(\beta\lambda_{t+1})$ in period t. (ii) Each sick old agent in period t, through health expenditure m_t, achieves health status ψm_t^{ψ}, which is lower than the healthy level 1. This reduction in utility is $\delta\omega(\ln 1 - \ln(\psi m_t^{\psi})) = -\delta\omega\ln(\psi m_t^{\psi})$ (cf. (3.7)). The second equality in (3.19) follows by substituting in the other optimality conditions.

In (3.20), since pollution P_t in period t is "inherited" (at rate $(1-\zeta)$) by the future periods (cf. (3.4)), the total marginal damage equals the infinite sum of the discounted marginal damage from period t on.

Proposition 12. *In the social planner's optimal solution (indicated by superscript $*$),*
(i) $c_t^{o,d*} = c_t^{o*}$: there is complete risk sharing for the old agents;
(ii) the abatement expenditure q_t^* satisfies

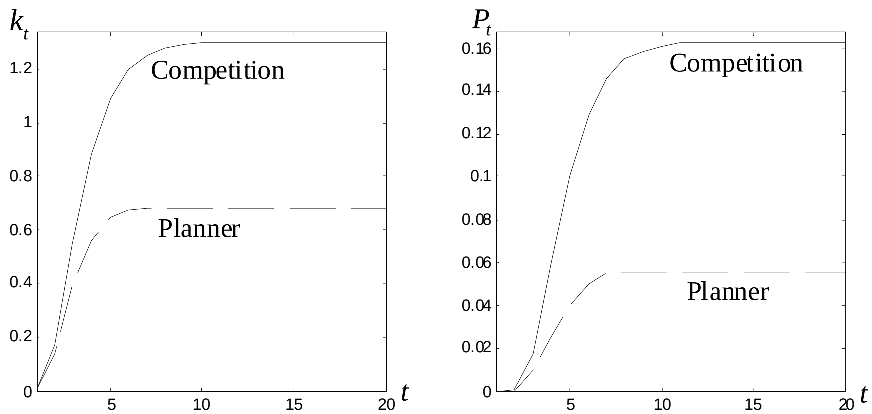
$$q_t^* \begin{cases} = G'^{-1}\left(1/\left(c_t^{y*}D_{t+1}^*\right)\right), & \text{if} \quad \frac{1}{c_t^{y*}} = G'\left(q_t^*\right)D_{t+1}^* \\ = G^{-1}\left(\rho k_t^* + (1-\zeta)P_t^*\right), & \text{if} \quad \frac{1}{c_t^{y*}} < G'\left(q_t^*\right)D_{t+1}^* \end{cases} \tag{3.21}$$

(iii) the steady state capital $\overline{k}^*$ is decreasing in the capital's pollution intensity ρ, and the steady state marginal damage of pollution $\overline{D}$. Further, $f'(\overline{k}^*) > 1/\beta$: $\overline{k}^*$ is lower than the modified golden rule level without pollution, $\hat{k}$.

Since the agents are risk averse, the social planner has incentive to provide complete risk sharing: an old agent has the same level of consumption regardless of his health status. Recall that in the market solution, health expenditure when sick ($m_t > 0$) implies that a sick old agent consumes less than a healthy old agent.

In period t, the marginal cost of abatement is given by the marginal utility of foregone consumption $1/c_t^y$. But one unit of period t abatement reduces period $t+1$ pollution by $G'(q_t)$ units, avoiding $G'(q_t)D_{t+1}$ units of pollution damages. The optimal condition for an interior q_t is thus $\frac{1}{c_t^{y*}} = G'(q_t^*)D_{t+1}^*$. Since $\lim_{q_t\to 0} G'(q_t) = \infty$, the optimal abatement q_t^* is strictly positive. However, it is possible that the marginal benefit is much larger than the marginal cost of q_t, and q_t takes the boundary values such that all current pollution is abated (so that $P_{t+1} = 0$). This is described by the second possibility in (3.21). Numerical simulation shows that it is possible for q_t to take interior or boundary values in the steady state or along the transition path.

It is intuitive that the steady state capital $\overline{k}^*$ is decreasing in the pollution intensity and in the damages of pollution. From Propositions 11 and 12(iii), $\overline{k}^*$ is lower than $\hat{k}$, the golden rule capital level without pollution. Thus, if an economy does not under-accumulate capital, or, if the steady state capital in the market equilibrium $\overline{k}$ is higher



(a) Path of capital (b) Path of pollution

FIGURE 3.6. Comparing market and socially optimal solutions

than $\hat{k}$, then $\overline{k} > \overline{k}^*$: the market equilibrium accumulates too much capital.

We continue the numerical example of Section 3.4.3, by setting the planner's discount rate, β, equal to the private agent's discount rate, δ; setting $\rho = 0.1$ to satisfy market solution $\sigma(\overline{P}) = 0.38$; and setting $G(q) = 0.3q^{0.8}$ to make pollution abatement expenditure to account for 1.69% of total output. Figure 3.6 compares the optimal pollution paths of the planner and market solutions. Evidently both the capital and pollution levels in the market equilibrium, are higher than those in the social optimum.

3.6 Policy Interventions

There are two main inefficiencies in the market equilibrium: the lack of consumption smoothing or risk sharing for the old, and the lack of mechanisms to internalize the pollution externality of capital. In this section, we consider policy interventions that address, directly or indirectly, the two inefficiencies. We first evaluate two common stabilization instruments, private health insurance and pay-as-you-go (PAYG) insurance akin to the U.S. Medicare system. We then evaluate the effects of a Pigouvian tax as well as a set of first best policies. To highlight the two inefficiencies rather than the dynamic inefficiency inherent in OLG models, we evaluate the impacts of the interventions on the steady state utility levels of an agent (cf. (3.7)) rather than on the discounted social utility in (3.17).

3.6.1 Health Insurance

We distinguish between two health insurance systems: under actuarially fair *private insurance*, the premiums paid by a young agent at period t are invested by private insurance companies and returned with interest at period $t+1$ to cover the medical expenses of this agent if he is sick; under the *PAYG insurance*, tax raised by young agents at period t is used to cover the health expenditures of the old agents in the same period t. The two systems have different impacts: private insurance can implement complete risk sharing but does not affect economy-wide capital accumulation; in contrast, PAYG insurance can reduce the capital stock but cannot guarantee complete risk sharing.

Private Health Insurance Suppose young agents can purchase health insurance that covers medical expenditures when old and sick. In each period, competitive, zero-cost insurance companies collect premiums from the young and invest the proceeds to cover medical expenses of the sick old in the following period. Specifically, insurance companies offer health insurance to the young in period t at the actuarially fair price, $\sigma(P_{t+1})/R_{t+1}$. By paying $\sigma(P_{t+1})/R_{t+1}$ units of consumption goods to the insurance company when young, an old agent verified to be in poor health receives one unit of health expenditure. An agent maximizes his life time utility (3.7) subject to the following budget constraints (cf. (3.8) - (3.10)):

$$s_t + c_t^y + \frac{\sigma(P_{t+1})}{R_{t+1}} x_{t+1} = w_t \tag{3.22}$$

$$c_{t+1}^o = s_t R_{t+1} \tag{3.23}$$

$$c_{t+1}^{o,d} + z_{t+1} = s_t R_{t+1} \tag{3.24}$$

$$m_{t+1} = x_{t+1} + z_{t+1}, \tag{3.25}$$

where x_{t+1} is the amount of insurance he purchases, z_{t+1} is health expenditure over and above his insurance coverage, and $m_{t+1} = x_{t+1} + z_{t+1}$, as before, is the total health expenditure. The optimal solutions are

$$z_{t+1} = 0 \tag{3.26}$$

$$\frac{c_{t+1}^o}{\delta c_t^y} = R_{t+1} \tag{3.27}$$

$$c_{t+1}^{o,d} = c_{t+1}^o = \frac{x_{t+1}}{\omega\psi} = \frac{m_{t+1}}{\omega\psi} \tag{3.28}$$

$$s_t = \theta_t^I w_t, \quad \text{with} \quad \theta_t^I \equiv \delta / \left[1 + \delta + \delta\omega\psi\sigma(P_{t+1}\right]. \tag{3.29}$$

From (3.26) and (3.28), the risk averse agent purchases full health coverage and achieves complete risk sharing: when old and sick, the insurance covers all of his expenditures and his consumption is not reduced. The agent's savings behavior is also changed by the availability of health insurance: as shown in (3.29), his savings rate θ_t^I is decreasing in pollution level P_{t+1}. This is in contrast to the benchmark market model without insurance: in (3.12), the savings rate θ_t is increasing in P_{t+1}. Intuitively, a higher pollution level leads to increased health risk and thus higher price of health insurance. Further, the agent's demand for health insurance is inelastic.[16] Consequently the total insurance expenditure (premium times x_{t+1}) increases, reducing the agent's savings.

Since the premiums collected by insurance companies are invested,[17] the total savings (or capital stock) of the economy consist of individual savings as well as the insurance premium:

$$k_{t+1} = s_t + \frac{\sigma(P_{t+1})}{R_{t+1}} x_{t+1} = \frac{\delta + \delta\omega\psi\sigma(P_{t+1})}{1 + \delta + \delta\omega\psi\sigma(P_{t+1})} w_t. \tag{3.30}$$

Comparing with (3.12), and noting that (3.13) and (3.14) determine the economy's pollution levels, we know the total savings are exactly the same as those in the the benchmark model without insurance. Private insurance does not affect the *total* capital accumulation, neither does it affect the consumption level of the young, c_t^y. Nevertheless, private insurance, by allowing complete risk sharing, improves the agents' utility levels (illustrated in Figure 3.7). In summary,

Proposition 13. *Actuarially fair private health insurance has no effect on the total capital accumulation in the economy, but can implement complete risk sharing, thereby raising the agents' utility levels.*

PAYG Health Insurance Suppose in period t, each young agent pays a lump-sum tax g_t and each sick old agent receives b_t in the form of government payments for health expenditures. *Ex ante* balancing of the government budget requires

$$\sigma(P_t) b_t = g_t. \tag{3.31}$$

[16]It is easy to check that the price elasticity of x_{t+1} satisfies $\left|\frac{\partial x_{t+1}}{\partial \sigma(P_{t+1})} \frac{\sigma(P_{t+1})}{x_{t+1}}\right| < 1$.

[17]Again, in a closed economy, the premiums are invested domestically.

An agent, taking g_t (and b_t) as given, maximizes his life time utility (3.7) subject to the following constraints:

$$\begin{aligned} s_t + c_t^y &= w_t - g_t && (3.32)\\ c_{t+1}^{o,d} + z_{t+1} &= s_t R_{t+1} && (3.33)\\ c_{t+1}^{o,h} &= s_t R_{t+1} && (3.34)\\ m_{t+1} &= z_{t+1} + b_{t+1} && (3.35) \end{aligned}$$

where z_{t+1} is the extra health expenditure over and above government insurance coverage.

We focus our analysis on the steady state of the economy, in which the health expenditure payment from government is constant: $b_t = b$.

Proposition 14. *In the (nontrivial) stable steady state,*

(i) PAYG health insurance reduces the capital stock of the economy: $\partial \overline{k}/\partial b < 0$.

(ii) If the steady state capital stock exceeds the golden rule capital level without pollution, i.e., if $\overline{k} > \widehat{k}$, *then each agent's steady state utility is increasing in* b.

PAYG insurance reduces the savings rate through two channels: the current tax g on the young reduces their disposable income, and future medical payment b reduces the need for precautionary savings. If there is over-accumulation of capital in the market economy, then PAYG insurance, by reducing savings, unambiguously improves the agents' life time utility levels.

An old agent's additional health expenditure z_{t+1} is decreasing in the level of insurance b_{t+1}. From (3.33) and (3.34), complete risk sharing (i.e., equal consumption levels for healthy and sick old agents) is achieved only when $z_{t+1} = 0$. A natural question to ask is whether it is optimal for the planner to choose a level of b so that PAYG insurance leads to complete risk sharing (at least in the steady state). The following proposition shows that this is not the case.

Proposition 15. *Suppose the government chooses the PAYG insurance level to maximize the steady state utility* $\overline{U}$. *At this optimal insurance level, there is incomplete risk sharing so that* $\overline{z} > 0$.

PAYG insurance serves two purposes: it helps share the risk and smooth out consumptions between the healthy and sick old, and by reducing the income of the young, it reduces the savings rate and thus the steady state capital. Generically, the existence of

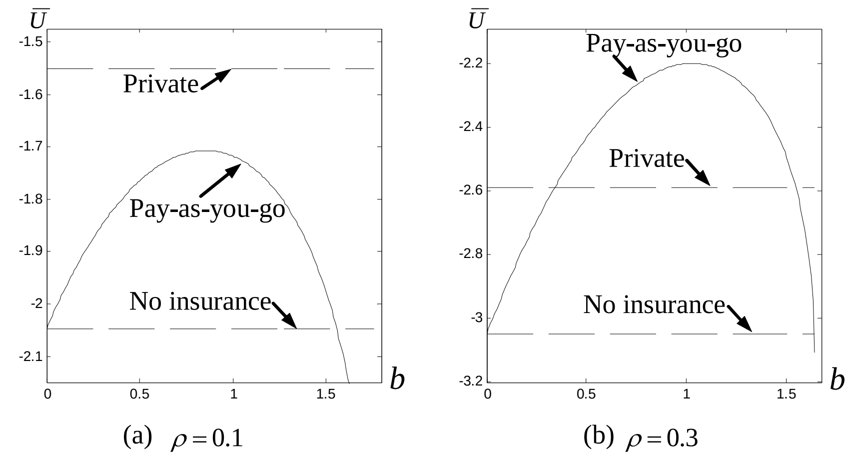


FIGURE 3.7. Comparing steady state utility in different insurance systems

the second objective means that the first objective of complete risk sharing is not fully achieved. Of course, if there is private insurance in addition to PAYG insurance, young agents will purchase enough private insurance to "fill the slack of the PAYG insurance," thereby obtaining complete risk sharing.

Figure 3.7 compares the steady state utilities of the benchmark market equilibrium, private health insurance system and PAYG health insurance system for different values of pollution intensity of capital, ρ. Offering private insurance always improves the utility levels, and the increase in utility is due entirely to risk sharing (and thus independent of ρ). Consistent with Proposition 14, under PAYG insurance, the steady state utility is increasing in coverage level b when b is low (i.e., when capital $\overline{k}$ is high). But as the coverage is further raised, steady state capital $\overline{k}$ is too low and further increases in b reduce utility. In fact, if b is too high, the PAYG utility can even be lower than the benchmark utility level.

When the capital's pollution intensity is low ($\rho = 0.1$), PAYG insurance is dominated by private insurance. Since capital is not too pollutive, the cost due to over-accumulation of capital under private insurance is more than compensated by the benefit from complete risk sharing. However, when ρ is high ($= 0.3$), PAYG insurance at certain coverage levels can dominate private insurance: ameliorating over-accumulation of capital becomes more important when capital is more pollution intensive. In this example, we can also verify that as long as a steady state equilibrium exists, the medical expenditure z is positive for both values of ρ: no complete risk sharing is ever achieved under PAYG insurance.[18]

3.6.2 Environmental Policies

We next consider the effects of a pollution tax. Suppose the government imposes a pollution tax τ_t in period t on the polluting firms. Then the firm's optimization problem is

$$\underset{K_t, L_t, q_t}{Max}\, F(K_t, L_t) - R_t K_t - w_t L_t - \tau_t [\rho K_t - G(q_t)] - q_t \tag{3.36}$$
$$s.t. \quad \rho K_t - G(q_t) \geq 0.$$

and the necessary conditions are (3.2) and

$$R_t \geq f'(k_t) - \rho\tau_t, \quad R_t = f'(k_t) - \rho\tau_t \quad \text{if} \quad G(q_t) < \rho k_t; \tag{3.37}$$
$$\tau_t G'(q_t) \geq 1, \ = 1 \text{ if } G(q_t) < \rho k_t \tag{3.38}$$

At interior solutions (where abatement q_t only removes part of the emissions ρk_t), (3.37) shows that the net return of capital is reduced by the tax penalty, and in (3.38), q_t is chosen to equate the marginal benefit of abatement (reduced tax expenditure), $\tau_t G'(q_t)$, with its marginal cost, 1. Substituting the wage rate in (3.2) and the reduced rent to capital in (3.37) to (3.36), we know the firm receives a positive profit of $\pi_t = \tau_t G(q_t) - q_t$, which is generated from the pollution abatement activity. Given that the ownership of the firm is defined by capital, each unit of capital "earns" a share of the profit, π_t / k_t. Thus the final return of capital is equal to the interest rate plus profit per unit of capital: $\widehat{R}_t = R_t + \pi_t / k_t$. We can show that even with the profit share, $\widehat{R}_t$ is still lower than the capital rate of return without the pollution tax, $\alpha A k_t^{\alpha-1}$.

The tax revenue, $\tau_t [\rho k_t - G(q_t)]$, can be redistributed to the agents in many ways. Since the pollution tax is levied on the firm owned by the old agents, we study a scenario where the tax revenue is rebated back to the old in a lump-sum manner. We later discuss the implications of other ways to redistribute the tax revenue.

Given the capital rate of return $\widehat{R}_t$ and the lump-sum rebate scheme, the young agent

[18] By De La Croix and Michel (2002), there exists a finite threshold $b^{\max} > 0$ such that for $0 < b < b^{\max}$, the dynamic system has two steady states $\underline{k}(b)$ and $\overline{k}(b)$ and for $b > b^{\max}$, no steady state equilibrium exists. The threshold $b^{\max}$ is determined by $\partial b / \partial \overline{k} = 0$, where $\partial b / \partial \overline{k}$ can be derived from (C.14) by the implicit function theorem. In Figure 3.7, $b^{\max} = 1.75$ for the case of $\rho = 0.1$ and $b^{\max} = 1.63$ for the case of $\rho = 0.3$.

maximizes (3.7) subject to (3.8) and

$$c_{t+1}^{o,d} + m_{t+1} = s_t \widehat{R}_{t+1} + \mu_{t+1} \quad (3.39)$$

$$c_{t+1}^{o,h} = s_t \widehat{R}_{t+1} + \mu_{t+1}, \quad (3.40)$$

where $\mu_{t+1} = \tau_{t+1}(\rho k_{t+1} - G(q_{t+1}))$ is the lump-sum subsidy specified above. In the optimal solution,

$$s_t = \frac{[\delta + \omega\psi\delta(P_{t+1})]\, w_t - \mu_{t+1}/\widehat{R}_{t+1}}{1 + \delta + \omega\psi\delta(P_{t+1})}. \quad (3.41)$$

The pollution tax lowers the savings rate through two channels: by reducing pollution level P_{t+1} and thus the health risk, and by raising the old age income by μ_{t+1}.[19] The first channel is akin to the "standard" effects of a Pigouvian tax, while the second channel is due to the distribution of the tax revenue, and is thus a "double dividend." Since the revenue from pollution taxes is used to subsidize the old, the pollution tax is effectively financing a social pension system, reducing the need for precautionary savings. In static models, double dividend is in general considered to be non-existent in general equilibrium settings, e.g. Bovenberg and De Mooj (1994) and William (2002, 2003). But our results indicate that in a dynamic general equilibrium setting, appropriate distribution of the tax revenue can reduce preexisting distortions of intertemporal allocation (in household savings), and as a result, the optimal tax level can be higher than Pigouvian tax (see Table 3.2). For the purpose of reducing precautionary savings, distributing the pollution tax revenue to the old and sick is likely to be more effective than transferring the revenue to the old, while returning the revenue to all agents (young and old) is likely to be less effective.

We continue our numerical example to compare the optimal pollution tax in a second best setting to the Pigouvian tax in the first best setting. The comparison is implemented at steady state. First, the steady state with pollution tax is given by

$$\overline{k} = \frac{\left[\delta + \omega\psi\delta(\overline{P})\right](1-\alpha)\, A\overline{k}^{\alpha} - \overline{\mu}/\overline{R}}{1 + \delta + \omega\psi\delta(\overline{P})} \quad (3.42)$$

$$\overline{P} = \frac{\rho\overline{k} - G(\overline{q})}{\zeta} \quad (3.43)$$

where $\overline{\mu}/\overline{R} = \tau\left[\rho\overline{k} - G(\overline{q})\right] / \left[f'\left(\overline{k}\right) - \rho\tau + \overline{\pi}/\overline{k}\right] = \left[\tau\rho\overline{k} - \tau G(\overline{q})\right] / \left[f'\left(\overline{k}\right) - \overline{q}\right]$. We

[19] Although the pollution tax reduces the capital rate of return, the lower return to savings does not directly reduce the savings rate due to the log-utility assumption, as indicated in Lemma 8.

TABLE 3.2. Comparing steady state solution between first and second best cases

	First best	Second best
τ	3.028	3.204
$\overline{U}$	-1.422	-1.929
$\overline{k}$	0.872	1.158
$\overline{P}$	0.004	0.014
$\overline{q}$	0.203	0.269

TABLE 3.3. Comparing steady state utility for different policy combinations

	Tax+Private	Private+PAYG	Tax+PAYG
$\rho = 0.1$	-1.4613	-1.4933	-1.5947
$\rho = 0.3$	-2.1677	-2.0864	-2.0844

set $\rho = 0.1$ following the previous numerical example, and Table 3.2 presents the results. Consistent with the literature, when there exists a double dividend, the optimal pollution tax in the second best setting is higher than the Pigouvian tax.

3.6.3 Combinations of Government Policies

Nor surprisingly, combinations of policies, properly designed, improve the agents' steady state utility compared with individual policies. Table 3.3 compares three pair combinations of the above discussed policies, where the policy levels are optimally chosen in each combination. Recall that, in Figure 3.7, private insurance dominates PAYG insurance for low pollution intensity $\rho = 0.1$ and is dominated by optimally chosen PAYG when $\rho = 0.3$. This welfare ordering is preserved when each insurance is combined with pollution tax. For both pollution intensity levels, combining pollution tax with the "right" kind of insurance welfare dominates the combination of the two kinds of insurance only, showing the importance of pollution tax.

To restore the first best, typically three policy instruments are needed to correct the three kinds of distortions in the economy, namely consumption smoothing, pollution externality, and dynamic inefficiency. Interestingly, the combination of pollution tax, private insurance and PAYG insurance cannot restore the first best. The reason is that under pollution tax, the profit generated from pollution abatement distorts the capital rate of return. That is, while correcting one distortion, the tax creates another distortion.

Another policy, e.g., a profit tax or intergenerational transfer (e.g., social security), is required to correct the newly created distortion.

3.7 Conclusion

In this paper, we study the pollution-economic growth nexus from the perspective of health and precautionary savings. We first establish empirical evidence based on the Chinese data that higher pollution levels is associated with high savings rates. Consistent with this phenomenon, we construct an OLG model in which agents save more in response to the higher future probability of getting sick due to higher pollution levels, and the increased savings in turn lead to more investment and thus more pollution. An economy might thus experience high economic growth rate coupled with high pollution levels. This economy may seem to be sustainable in terms of economic growth, but the welfare level is not sustainable due to the increased pollution.

Our work points to the fact that health, savings, pollution and growth are intertwined and evolve endogenously and interconnectedly in a growing economy; as such, policy interventions in one arena necessarily spillover into others. We consider three kinds of policy interventions to "break" the pollution-growth-pollution cycle: private insurance, pay-as-you-go insurance, and tax on pollution. In the dynamic general equilibrium setting, a stabilization policy (PAYG insurance), properly designed, can influence the pollution level and thus have environmental effects. Conversely, an environmental policy also affects the savings rate and health risks, and thus have stabilization implications. All three kinds of policies contribute to correcting the distortions in the economy: private insurance achieves full risk sharing but does not reduce the total investment or the pollution level; PAYG insurance reduces investment and pollution but can only achieve partial risk sharing; pollution tax reduces pollution and savings rate, but causes an additional distortion in capital rate of return. An additional policy instrument is needed to restore the first best.

The stabilization effect of the pollution tax partly depends on how the tax revenue is distributed. This effect arises only in a dynamic setting. That is, even when double dividends do not exist in a static setting, they can still arise in a dynamic setting through affecting the savings behavior.

The environmental objectives are best served in a system with intergenerational transfers from the young to the old (especially the old who are sick). Pay-as-you-go schemes (at properly chosen levels) are more desirable than schemes with private accounts (e.g., private insurance or private retirement accounts). But given that PAYG schemes can never achieve full risk sharing, they should be supplemented by private health insurance.

Given the existence of (dynamic) double dividends of the pollution tax, these nations should not only impose pollution control measures, but if they choose to impose a pollution tax, the tax rate should also be higher than the Pigouvian level. Further, the tax revenue should be redistributed to serve a "stabilization purpose". The most desirable approach is to redistribute the revenue to the old and sick, e.g., as contributions to premiums in a PAYG scheme.

The findings in this paper have important implications for developing nations that are experiencing rapid economic growth and environmental degradation, such as China and India. There have been intense debates about the sustainability of the (near) double digit growth rates due to the intensive exploitation of natural resources and the environment. Our paper suggests that even when these growth rates are sustainable, they may not be desirable from a welfare standpoint due to the increased health expenditures. More importantly, the lack of adequate health care and social security institutions might have inadvertently contributed to the pollution-growth-pollution cycle. This observation highlights the urgency of developing such institutions, and the necessity of incorporating environmental objectives in doing so.

Chapter 4

Optimal Education Policies under Endogenous Borrowing Constraints

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4.1 Abstract

When young students face exogenous borrowing constraints (incomplete markets) on education loans, the simultaneous establishment of a education subsidy and an old-age pension has been shown to restore the complete market allocation (Boldrin and Montes, 2005). If the borrowing constraint is endogenous, owing to limited commitment of repayment and inalienability of future human capital (as in Kehoe and Levine, 1993), Andolfatto and Gervais (2006) by means of an example, argue that the education-subsidy-cum-pension scheme distorts the credit market, and hence, fails to restore the complete market allocation. In this paper, I show that the complete market allocation can be achieved even with endogenous borrowing constraints. The result broadens the rationale for a two-armed (education and pension) welfare state to a much wider class of economies.

4.2 Introduction

In growth theory, human capital is deemed an important engine of economic growth (e.g., Lucas, 1988, and Azariadis and Drazen, 1990). [1] More generally, it is beyond controversy that education has a profound beneficial effect on the overall performance of an economy. Yet in most countries, students, especially those from poor families, are generally short of funds for educational investments. Why? Due to the inalienability of human capital, future labor income cannot be collateralized. As such, credit markets severely restrict

[1] The cross-country estimation by Barro and Sala-i-Martin (P524, 2003) shows that a 1.3 year increase in male upper–primary-level schooling raises the growth rate by 0.5 percent.

any borrowing against future human capital for education purposes. [2] Too little human capital is generated, and this constrains overall economic performance. A challenge for development theory emerges from this discussion. Given imperfect credit markets, can public policy restore human capital investment to socially optimal levels?

At first blush, it may appear that a carefully-chosen public subsidy to education could ensure optimal accumulation of human capital. In a recent, important paper, Boldrin and Montes (2005) show that this is generally not enough. They present a three-period overlapping generation model in which the young need to borrow to finance education, middle-aged agents are net lenders, and no borrowing is possible (incomplete markets). The market outcome in this case is clearly inefficient. Would a policy that taxes the middle-aged and makes education-linked transfers to the young work? Boldrin and Montes (2005) show that, while such a policy may improve matters, it may not replicate the complete market allocation. Moreover, such a policy may never get off the ground because the initial middle-aged would be hurt – they would pay into the system having never received a subsidy from the current old. They go on to show that if credit markets are missing, i.e., people are not allowed to borrow or lend, the only way to restore the efficiency is "establishing publicly financed education and pay-as-you-go pensions simultaneously, and by linking the two flows of payment via the market interest rate". By their setup, the joint institutional arrangements perfectly substitute the missing credit market and therefore can replicate the complete market allocation of human capital investment. Their study provides a rationale for the "cradle to grave" policies that are widely observed (Andolfatto and Gervais, 2006).

Boldrin and Montes (2005), no doubt, provides a deep insight into the welfare state as it pertains to education and pensions; however, it is fair to say that their treatment of the imperfection in the credit market is somewhat arbitrary. They simply assume non-existence of a credit market and impose a zero borrowing limit on the young. Much of the work in the literature on credit market imperfections has focused on relaxing the zero borrowing limit. Human capital investment subject to exogenous borrowing constraints has been studied in papers such as de Gregorio (1996) and Cartiglia (1997). More recently, motivated by Kehoe and Levine (1993), recent studies, like Lochner and Mongo-Naranjo

[2] The impact of borrowing constraints on education may not be so evident in high-income countries, such as USA (Cameron and Heckman, 2001; Cameron and Taber, 2004) but is definitely important for poorer countries. Based on cross-country regression analyses, De Gregorio (1996) and Flug et al. (1998) have shown that borrowing constraints limit the education investment. Jacoby (1994), by using household data from Peru, presents similar evidence that children withdraw from school earlier if their family is borrowing constrained.

(2002), Andolfatto and Gervais (2006) and de la Croix and Michel (2007), have begun to introduce endogenous borrowing constraints and examine their role in accumulation of human capital. The framework proposed by Kehoe and Levine (1993) and extended in the lifecycle model by Azariadis and Lambertini (2003) has become the de-facto benchmark for analyzing borrowing constraints. In that setup, the borrowing limit arises because the borrower cannot commit to repaying her loan, and if she defaults, the creditor can seize tangible assets but not her private inalienable endowments, such as human capital and government entitlements. Therefore, the only punishment for (or the opportunity cost of) defaulting is being excluded from the credit market for the rest of one's life, and as a consequence, being unable to smoothen consumption. Under perfect information, lenders would set the borrowing limit at the amount where cost and benefit of default are balanced. Hence, any loan less than this borrowing limit is in the borrower's interest to repay, and there is no default in equilibrium.

Evidently, as a natural extension of Boldrin and Montes (2005), one may ask: in the presence of endogenous borrowing constraints, would the Boldrin-Montes education-pension package replicate the complete market allocation? Andolfatto and Gervais (2006) take up this question and demonstrate the possibility that intergenerational transfer policies tighten the borrowing constraint and leave less resources for the young to invest in human capital. The intuition is that more education subsidies means more tax on the middle-aged and bigger pensions for the old, both of which reduce the need for consumption smoothing, increasing the incentive to default, and resulting in a further-tightened borrowing limit. Andolfatto and Gervais (2006) conclude that there does not exist optimal intergenerational policies with positive education subsidy that replicate the complete market allocation, for "the government subsidy (in education) does not compensate for the contraction in private lending".

It is useful to point out that the sharp conclusion of Andolfatto and Gervais (2006) relies entirely on a numerical example, and that example turns out to be somewhat of a special case as this paper shows. It is shown here that, depending on the model's parameters, the upshots of both Andolfatto and Gervais (2006) and Boldrin and Montes (2005) could be correct. How could this be? Recall from above that as the education subsidy increases, an individual's income profile becomes flatter, and consequently, the borrowing limit, as well as the educational investment of a borrowing-constrained individual falls, until the borrowing limit hits zero. At this level of the subsidy, the private market for education loans is completely choked off. If the subsidy is increased further, the borrowing limit remains at zero, and so the entirety of the educational investment is being funded by the subsidy. In particular, the subsidy can rise to the point at which educational investment by the young exactly equals its level in the complete markets case. Note though, that driving the borrowing limit to zero implies optimal savings of the middle-aged agent are forced to bind at zero precluding any consumption smoothing. It is evident that to replicate the complete market allocation, the "right" subsidy must achieve optimal educational investment and consumption smoothing simultaneously. For this to happen, total resources available for educational investment must remain larger than its level in the complete markets case *before* the borrowing limit hits zero. Below, I derive sufficient conditions for the existence of such an optimal subsidy. If the parametric specification of the model satisfies these conditions, the conclusion of Boldrin and Montes (2005) would extend to economies with endogenous borrowing constraints. If they do not hold, it is possible that the negative result in Andolfatto and Gervais (2006) would then apply.

The rest of the paper is organized as the follows. The complete market solutions are provided in section 4.3. The endogenous borrowing limit is determined in section 4.4. Section 4.5 examines the optimal intergenerational policies in the case of exogenous interest rate. Section 4.6 concludes. Proofs of all results are contained in the appendices at the end of the paper.

4.3 Complete markets economy

The model closely follows those described in Boldrin and Montes (2005) and Andolfatto and Gervais (2006). Consider a economy consisting of an infinite sequence of three-period lived overlapping generations, an initial old generation and an initial middle-aged generation. In each generation, there is a continuum of identical members of measure one. Each agent is born with an endowment profile $(\omega^y, \omega^m, \omega^o)$. She invests in human capital when young, works and receives return on that education investment during middle-age and is retired when old. As in Boldrin and Montes (2005), I assume agents draw utility from consumption at middle-age (c_t^m) and old age $\left(c_{t+1}^o\right)$. The utility function of an agent born at period $t-1$ is

$$u\left(c_t^m\right) + \beta u\left(c_{t+1}^o\right) \tag{4.1}$$

where β is the subjective discount factor and $u(\cdot)$ is a strictly increasing, concave function and twice continuously differentiable.

When young, an agent invests $x_{t-1} = \omega^y + b_{t-1}$ in human capital (there is no physical capital), in which b_{t-1} is savings if $b_{t-1} < 0$ and borrowings if $b_{t-1} > 0$. I assume $\omega^y = 0$

implying that young agents will need to borrow to finance their education. Middle-aged agents work and earn consumption goods $f(x_{t-1})$ where $f(x_{t-1})$ is the return on their prior education investment and f is a strictly increasing, concave function with $f(0) = 0$. With complete markets, each agent commits to repaying the loan. Her lifecycle budget constraints are

$$
\begin{aligned}
x_{t-1} &= \omega^y + b_{t-1}, && (4.2)\\
c_t^m + s_t &= \omega^m + f(x_{t-1}) - R_t b_{t-1}, && (4.3)\\
c_{t+1}^o &= \omega^o + R_{t+1} s_t, \text{ and} && (4.4)\\
0 &\leq b_{t-1} \leq b_{t-1}^{\max}. && (4.5)
\end{aligned}
$$

Here R_t is the interest rate between $t-1$ and t, s_t is the savings of middle-aged agent, and $b_{t-1}^{\max}$ is the upper bound of the loan that young can borrow and is defined by the following equation

$$\omega^m + f\left(b_{t-1}^{\max} + \omega^y\right) - R_t b_{t-1}^{\max} = 0. \qquad (4.6)$$

For any borrowing $b_{t-1} > b_{t-1}^{\max}$, $\omega^m + f(x_{t-1}) - R_t b_{t-1} < 0$, i.e., the net income of the middle-aged agent is negative.

The first order conditions for the agent's problem are

$$
\begin{aligned}
\frac{u'\left(c_t^{m*}\right)}{u'\left(c_{t+1}^{o*}\right)} &= \beta R_{t+1} && (4.7)\\
f'\left(x_{t-1}^*\right) &= R_t && (4.8)
\end{aligned}
$$

where superscript * denotes the complete market solutions. Equation (4.7) equates marginal rate of substitution of consumption to the discounted interest rate. Equation (4.8) implies that marginal return from investing in human capital should equal the marginal cost of the loan. By (4.7), we can obtain the explicit solutions of x_{t-1}^* and b_{t-1}^* :

$$
\begin{aligned}
x_{t-1}^* &= f'^{-1}(R_t), && (4.9)\\
b_{t-1}^* &= f'^{-1}(R_t) - \omega^y. && (4.10)
\end{aligned}
$$

Finally, in a closed economy, the interest rate is determined by the following general equilibrium condition

$$s_t^*\left(R_t^*, R_{t+1}^*\right) = b_t^*\left(R_{t+1}^*\right) \qquad (4.11)$$

which, for given initial interest rates (R_{-1}, R_0), clears the credit market for the initial debt. The first-order difference equation (4.11) characterizes the dynamics of the economy.

4.4 Borrowing-constrained economy

In this section, I study an economy in which agents cannot commit to repay their loans and their ability to borrow against future income is limited due by the absence of commitment.

As in Kehoe and Levine (1993), all information is public, and in the event of default, the affected creditors cannot seize the individual's private endowments or education returns, but can appropriate her current and future assets. The only punishment creditors can impose is to keep the defaulter out of credit market for the rest of her life. For borrowers, the cost of default is the foregone lifetime gains from trading in the credit market. Since all information, including the default, is public, creditors allow agents to borrow up to a limit which is in her interests to repay, i.e. for all loan amounts less than that limit, the benefit from trading in the credit market is bigger than the cost of autarkic consumption. As such, default never occurs in equilibrium.

4.4.1 The Basics

Since all agents borrow when young, it is the choice of the middle-aged agent to default or not. If she contemplates repaying the loan, she faces an optimization problem identical to that in the complete markets economy. Otherwise, she will be excluded from credit market and consume

$$c_t^{m,d} = \omega^m + f\left(b_{t-1} + \omega^y\right) \tag{4.12}$$

$$c_{t+1}^{o,d} = \omega^o \tag{4.13}$$

where the superscript d denotes the allocation in the case of default.

For creditors, the optimal lending decision for agents born at $t-1$ is the solution to the problem that maximizes (4.1) subject to the budget constraints (4.2)−(4.5) and the following individual rationality constraints (IRC)

$$s_t \geq 0 \qquad \text{IRC (1)}$$

$$V_t^m\left(\omega, b_{t-1}\right) \geq u\left[\omega^m + f\left(b_{t-1} + \omega^y\right)\right] + \beta u\left(\omega^o\right), \qquad \text{IRC (2)}$$

where

$$V_t^m(\omega, b_{t-1}) \equiv \max_{s_t} \left\{u\left[\omega^m + f\left(b_{t-1} + \omega^y\right) - R_t b_{t-1} - s_t\right] + \beta u\left(s_t R_{t+1} + \omega^o\right)\right\}$$

is the value function of the middle-aged agent who repays the loan and can access the credit market. IRC1 implies she cannot borrow, for participation in the credit market has no value to her during old age and hence, she will never repay the debt. IRC2 implies that creditors should always offer a loan that makes the borrower prefer repayment to default.

Now, consider the optimization problem of an agent in the borrowing-constrained economy. She takes the borrowing limit $\bar{b}_{t-1}$ as exogenous. If $\bar{b}_{t-1} < b_{t-1}^*$, she is borrowing constrained. Otherwise, her debt constraint is slack. If the latter is the case, the optimality conditions are the same as those in the case of the complete market economy. Otherwise, her first order conditions are

$$b_{t-1}^c = \bar{b}_{t-1} \tag{4.14}$$

$$f'\left(x_{t-1}^c\right) = R_t + \frac{\lambda_t}{u'\left(c_t^{m,c}\right)} \geq R_t \tag{4.15}$$

$$\frac{u'\left(c_t^{m,c}\right)}{u'\left(c_{t+1}^{o,c}\right)} \geq \beta R_{t+1}, \; = \text{ if } s_t^c > 0 \tag{4.16}$$

where the superscript c denotes the optimal solution of individual in the constrained market, and $\lambda_t > 0$ is the Lagrangian multiplier of the borrowing constraint $b_{t-1} \leq \bar{b}_{t-1}$. Equation (4.14) implies that, $x_{t-1}^c < x_{t-1}^*$, human capital is under-invested in the imperfect credit market. Equation (4.15) states that the marginal return from human capital investment is higher than the interest rate. Due to the borrowing constraints, the gains from the investment opportunity cannot be exhausted.

4.4.2 Endogenous Borrowing Limits

In the following, I examine the individual rationality constraints, IRC1-IRC2, to determine the aforediscussed borrowing limit. First, I characterize the conditions for IRC1, the non-negativity constraint on savings.

Proposition 16. *An agent born at $t-1$ is borrowing-constrained at both period $t-1$ and period t with $\bar{b}_{t-1} = 0$ if and only if $u'\left[\omega^m + f\left(\omega^y\right)\right]/u'\left(\omega^o\right) > \beta R_{t+1}$.*

If condition $u'\left[\omega^m + f\left(\omega^y\right)\right]/u'\left(\omega^o\right) > \beta R_{t+1}$ holds, the middle-aged agent has no incentive to save even if she does not incur any debt when young. Clearly, in this case, the borrowing limit is zero. On the flip side, when $u'[\omega^m + f(\omega^y)]/u'(\omega^o) < \beta R_{t+1}$, the optimal saving of a middle-aged agent without any prior borrowing is positive, i.e. $s_t^c > 0$. In this case, the borrowing limit is positive, i.e. $\overline{b}_{t-1} > 0$; henceforth I assume, $u'[\omega^m + f(\omega^y)]/u'(\omega^o) < \beta R_{t+1}$ holds which, in turn, guarantees IRC1.[3]

Given this assumption, I proceed to characterize the conditions for the non-default constraint IRC2 and determine the borrowing limit.

Proposition 17. *Denote*

$$H \equiv V_t^m(\omega, b_{t-1}) - u[\omega^m + f(b_{t-1} + \omega^y)] - \beta u(\omega^o) \tag{4.17}$$

(1) the borrowing limit $\overline{b}_{t-1}(R_t, R_{t+1})$ satisfies $H(\overline{b}_{t-1}) = 0$. If there exist multiple solutions, $\overline{b}_{t-1}$ is equal to the smallest one.

(2) at $\overline{b}_{t-1}$,

$$\frac{\partial H}{\partial b_{t-1}}|_{b_{t-1}=\overline{b}_{t-1}} < 0. \tag{4.18}$$

Proposition 17 argues that the borrowing limit, $\overline{b}_{t-1}$, should be determined at the point where the benefit of debt default equals its cost; from (4.18), it is clear that a marginal increase in the borrowing limit would break that balance, and thus violate IRC2.

In equilibrium, creditors would only allow young agents to borrow up to $\overline{b}_{t-1}$, and in their self interest, borrowers would repay the loan in the next period. Unlike the traditional credit rationing models based on asymmetric information between borrowers and lenders, this framework allows the existence of credit rationing even when all information is public; moreover it removes default in equilibrium.

I collect some useful properties of borrowing limit $\overline{b}_{t-1}(R_t, R_{t+1})$,

Corollary 18. *Given a sequence of interest rated $\{R_t\}$,*

(1) the borrowing limit of youth $\overline{b}_{t-1}$ is increasing in R_{t+1} and decreasing in R_t.

(2) the borrowing limit of youth $\overline{b}_{t-1}$ is increasing in ω^y and ω^m, and decreasing in ω^o.

[3] Note that the condition $u'[\omega^m + f(\omega^y)]/u'(\omega^o) < \beta R_{t+1}$ can be satisfied by parameter specifications that favor savings, such as sufficiently-low ratio of ω^o to ω^m, high β or high intertemporal elasticity of substitution. Intuitively if agent's savings incentive is sufficiently strong at middle age, her cost of defaulting on youthful debt — being excluded from the credit market — would be high and realizing that, creditors would lend to her. Moreover, as will be shown below, the borrowing limit as well as agent's defaulting cost is increasing in the incentive for savings at middle age.

The relationship between borrowing limit and the interest rate, discussed in part (1) of the corollary above, is the same as that in Azariadis and Lambertini (2003) and Croix and Michel (2007).[4] Part (2) of the corollary is crucial to the subsequent analysis because, as will be evident shortly, the intergenerational transfer policies proposed by Boldrin and Montes (2005) may be equivalently expressed as changes in the individual's endowment profile ω^i, $i = \{y, m, o\}$. Since the incentive for middle-aged agents to participate in the credit market is to smooth post-youth consumption, as post-youth endowments flatten, i.e., ω^m decreases or ω^o increases, agents gain less from trade raising their incentive to default, which results in tightened borrowing limits. Of course, increasing ω^y has the same effect as increasing ω^m.[5]

In sum, an agent born at $t-1$ is (not) borrowing-constrained if and only if

$$\bar{b}_{t-1}\left(R_t, R_{t+1}\right) < (\geq)\, b^*_{t-1}\left(R_t\right);$$

optimal borrowing of the young is given by $b^c_{t-1} = \min\left\{\bar{b}_{t-1}\left(R_t, R_{t+1}\right), b^*_{t-1}\left(R_t\right)\right\}$. The interest rate in the borrowing-constrained economy is determined from the following market clearing condition,

$$b^c_t = \min\left\{\bar{b}_t\left(R_{t+1}, R_{t+2}\right), b^*_t\left(R_{t+1}\right)\right\} = s^c_t\left(R_t, R_{t+1}\right) \tag{4.19}$$

given initial interest rates (R_{-1}, R_0). Given the stated goals of the paper, I assume $\bar{b}_{t-1}\left(R_t, R_{t+1}\right) < b^*_{t-1}\left(R_t\right)$ in what follows.

4.5 Policies

To reduce the burden of notation, I will examine optimal policies *at* the (borrowing-constrained) steady state. The argument also applies to the transitional path. In addition, in order to compare with results from Andolfatto and Gervais (2006), I begin the discussion in a small open economy facing an (exogenous) interest rate R. Such a sim-

[4] Intuitively, if current interest rate R_t is high, the debt size carried from youth to middle age is large. As the consequence, the income profile becomes flatter and middle-aged agent has less incentive to smooth consumption and reimburse her loan, leading to tightened borrowing limit. On the contrary, when expected future interest rate R_{t+1} is high, return from participating in credit market becomes high and therefore repaying loans in middle age becomes more attractive.

[5] It is the force of the savings incentive that determines the borrowing limit. It is straightforward to check that the borrowing limit could be raised if discount factor β is higher, human capital more productive or the intertemporal elasticity of substitution is higher. In all these cases, due to the high valuation of consumption smoothing, agents have a strong incentive to repay their loans.

plified setting helps obtain sharp results. The closed-economy case (endogenous interest rate) is examined in the appendix.

4.5.1 Small open economy

Consider a lump-sum transfer scheme (τ^y, τ^m, τ^o). As discussed in Andolfatto and Gervais (2006), a policy that tries to replicate the complete market solution must satisfy the government budget constraint,

$$\tau^y + \tau^m + \tau^o = 0 \tag{4.20}$$

and keep the present value lifecycle budget constraint of the agent unchanged

$$\tau^y + \frac{\tau^m}{R} + \frac{\tau^o}{R^2} = 0. \tag{4.21}$$

Therefore, the only possible choice of policy scheme that can restore the complete market solution is

$$\tau^m = -(1+R)\tau^y \tag{4.22}$$

$$\tau^o = R\tau^y \tag{4.23}$$

which is the same as the optimal policy in Boldrin and Montes (2005). Policy 3-tuples (τ^y, τ^m, τ^o) is collapsed to a one-tuple policy choice, τ^y. As will be shown below, any policy that replicates the complete market allocation, must additionally achieve optimal education investment and optimal consumption smoothing. Generically, one policy tool cannot achieve two goals, which may explain why, for some parameterization, optimal policy with $\tau^y > 0$ could be non-existent (as demonstrated in Andolfatto and Gervais, 2006).

Since a lump-sum transfer is equivalent to a re-arrangement of endowment profiles, all analyses reported in the previous section can be applied to this section with ω^i being replaced by $\omega^{i\prime} = \omega^i + \tau^i$, for $i = \{y, m, o\}$. The intergenerational transfer policy encourages the middle-aged agent to lend more generously to the young with a commitment of pay back in the form of a pension when old. When the borrowing constraint is exogenous, those transfers can perfectly substitute the missing credit market as in Boldrin and Montes (2003). When the borrowing constraint is endogenous, the government in trying to use intergenerational transfers to substitute the missing credit market distorts margins in the credit market, in particular, the level of the borrowing limit.

Using Corollary 18, the effect of government policy on the borrowing limit can be expressed as

$$\frac{\partial \bar{b}}{\partial \tau^{y}} = \frac{\partial \bar{b}}{\partial \omega^{y\prime}} - (1+R)\frac{\partial \bar{b}}{\partial \omega^{m\prime}} + R\frac{\partial \bar{b}}{\partial \omega^{o\prime}}$$
$$= \left[f'\left(\bar{b}+\omega^{y\prime}+\tau^{y}\right) - 1 - R\right]\frac{\partial \bar{b}}{\partial \omega^{m\prime}} + R\frac{\partial \bar{b}}{\partial \omega^{o\prime}}.$$

Setting $\omega^{y}=0$ and substituting (C.26) and (C.27), we can obtain

$$\frac{\partial \bar{b}}{\partial \tau^{y}} = \frac{\left[f'\left(\bar{b}+\tau^{y}\right)-1-R\right]\left[u'\left(c^{m,c}\right)-u'\left(c^{m,d}\right)\right]+\beta R\left[u'\left(c^{o,c}\right)-u'\left(c^{o,d}\right)\right]}{-\left[f'\left(\bar{b}+\tau^{y}\right)-R\right]u'\left(c^{m,c}\right)+f'\left(\bar{b}+\tau^{y}\right)u'\left(c^{m,d}\right)} \tag{4.24}$$

Equation (4.24) is the main analytical result of this paper and from it, I can derive important implications missed in the simulation example of Andolfatto and Gervais (2006).

A most important finding from equation (4.24) is that, even with endogenous borrowing constraints, the main argument of Boldrin and Montes (2005) could remain valid. Andolfatto and Gervais (2006) make a crowding-out type argument to argue no $\tau^{y} \geq 0$ implements the complete market allocation: "a one dollar education subsidy may well lead to a reduction in private credit by more than one dollar, leaving the young with less resources than prior to the intervention". Mathematically, that says

$$\frac{\partial x}{\partial \tau^{y}} = \frac{\partial \bar{b}}{\partial \tau^{y}} + 1 \leq 0 \text{ or } \frac{\partial \bar{b}}{\partial \tau^{y}} \leq -1.$$

They show that, as τ^{y} increases from zero, the income profile becomes flatter, and consequently, the borrowing limit, as well as human capital investment, is monotonically decreasing until $\bar{b}=0$. At this level of τ^{y}, the private market for education loans is completely choked off. If τ^{y} is increased further, the borrowing limit remains at zero, and so the entirety of the human capital investment is funded by the subsidy. In this manner, τ^{y} can increase until $\tau^{y}=x^{*}$.[6] Note though, that driving $\bar{b}$ to 0 implies optimal savings of the middle-aged agent are forced to bind at zero precluding any consumption smoothing.[7] It is evident that to replicate the complete market allocation, the "right" $\tau^{y*} \geq 0$ needs to succeed in both dimensions: achieve optimal education investment and consumption smoothing simultaneously. Alternatively, if τ^{y} increases from zero, total resources available for human capital investment, $\bar{b}\left(\tau^{y}\right)+\tau^{y}$, must remain larger than x^{*} *before* the borrowing limit decreases to zero. Below, I will derive sufficient conditions for

[6]It is easy to check that $\frac{\partial \bar{b}}{\partial \tau^{y}}\big|_{\bar{b}=0}=0$. Hence borrowing limit has no response to government policy when it already zero.

[7]Recall that $\bar{b}$ is determined by $H=0$ in Proposition 17. If $\bar{b}=0$, obviously $s_{t}=0$.

the existence and non-existence of optimal $\tau^{y*} \geq 0$.

Define $\widehat{\tau}^y$ by

$$\frac{u'(\omega^{m\prime})}{u'(\omega^{o\prime})} = \frac{u'\left[\omega^m - (1+R)\widehat{\tau}^y + f(\widehat{\tau}^y)\right]}{u'(\omega^o + R\widehat{\tau}^y)} = \beta R \tag{4.25}$$

Note that $\widehat{\tau}^y$ adjusts the income endowment to a level that agent carrying no youthful debt prefers autarky at middle age. Then we can apply Proposition 16 and conclude that for any $\tau^y \geq \widehat{\tau}^y$, left hand side of (4.25) would be greater than βR, leading to zero borrowing limit and binding savings. Therefore the consumption smoothing condition reads $\tau^{y*} \leq \widehat{\tau}^y$, and the existence of an optimal subsidy $\tau^{y*} \geq 0$ need to satisfy $\bar{b}(\tau^{y*}) + \tau^{y*} \geq x^*$ and $\tau^{y*} \leq \widehat{\tau}^y$ simultaneously. These two conditions ensures the lump-sum transfers provide enough education funding for youth and substitute the missed credit market without squeezing it.

We note that, in the case of non-existence of optimal $\tau^{y*} \geq 0$, the increase of education subsidy cannot compensate the drop of borrowing limit, i.e. $\partial\bar{b}/\partial\tau^y \leq -1$, and therefore resource condition can only be satisfied when consumption smoothing condition is violated, i.e. $\tau^y > \widehat{\tau}^y$. However, $\partial\bar{b}/\partial\tau^y \leq -1$ is not always the case. In the extreme, it can be shown that $\partial\bar{b}/\partial\tau^y > -1$ for all $\tau^y \geq 0$. By equation (4.24), it is easy to check $\partial\left(\partial\bar{b}/\partial\tau^y\right)/\partial\left[f'\left(\bar{b}+\tau^y\right)\right] > 0$. Since in the borrowing-constrained economy $f'\left(\bar{b}+\tau^y\right) \geq R$, a low bound of $\partial\bar{b}/\partial\tau^y$ can be derived by substituting $f'\left(\bar{b}+\tau^y\right) = R$ in (4.24)

$$\frac{\partial\bar{b}}{\partial\tau^y} \geq \frac{1}{R} - \beta\frac{u'(\omega^o + R\tau^y)}{u'\left[\omega^m - (1+R)\tau^y + f\left(\bar{b}+\tau^y\right)\right]} > \frac{1}{R} - \beta\frac{u'(\omega^o)}{u'\left[\omega^m + f(x^*)\right]} \tag{4.26}$$

from which sufficient conditions on ω or β can be derived to ensure $\partial\bar{b}/\partial\tau^y > -1$ for all $\tau^y \geq 0$. As long as $\partial\bar{b}/\partial\tau^y \leq -1$ does not always hold, existence of optimal $\tau^{y*} \geq 0$ is possible.

I go on to derive sufficient condition on ω^m to guarantee existence of optimal $\tau^{y*} \geq 0$. Similar argument can apply for other parameters. First we need to characterize the upper bound of ω^m, which is define by $\bar{b}(\varpi^m) = x^*$, i.e.

$$\underset{s}{Max}\left\{u\left[\varpi^m + f(x^*) - Rx^* - s\right] + \beta u(w^o + sR)\right\} - u\left[\varpi^m + f(x^*)\right] - \beta u(w^o) = 0 \tag{4.27}$$

According to Corollary 18 borrowing limit is increasing in ϖ^m. The above definition implies that for any $\omega^m \geq \varpi^m$, the borrowing constraint is relaxed by $\bar{b}(\omega^m) > x^*$. For all meaningful discussion, ω^m should be less than ϖ^m such that the economy is initially borrowing constrained. Next, I define $\widehat{\omega}^m$ by

$$\frac{u'\left[\widehat{\omega}^m - (1+R)x^* + f\left(x^*\right)\right]}{u'\left(\omega^o + Rx^*\right)} = \beta R. \tag{4.28}$$

Comparing (4.25) to (4.28), we can learn that $\widehat{\omega}^m$ is the endowment level that equates $\widehat{\tau}^y$ to x^*. As presented in the following Proposition, $\widehat{\omega}^m$ is the threshold value for the existence of optimal $\tau^{y*} \geq 0$.

Proposition 19. *(1) Suppose $\overline{\omega}^m \geq \widehat{\omega}^m$, optimal $\tau^{y*} \geq 0$ replicating complete market solutions exists if and only if $\omega^m \in \left[\widehat{\omega}^m, \overline{\omega}^m\right)$.*

(2) If $\overline{\omega}^m < \widehat{\omega}^m$, there does not exist optimal $\tau^{y*} \geq 0$.

Because ω^m are ω^o are symmetric, similar conditions can be derived for ω^o which is required to be low enough. Furthermore, since by (4.25) $\widehat{\tau}^y$ is increasing in β, we can define $\overline{\beta}$ and $\widehat{\beta}$ in the same way such that optimal $\widehat{\tau}^y \geq 0$ exists if and only if $\beta \in \left[\widehat{\beta}, \min\left\{\overline{\beta}, 1\right\}\right)$ and $\widehat{\beta} < \overline{\beta}$.

Now from Proposition 19 we can conclude that if and only if agents have sufficiently high incentive to smooth consumption, does there exist an optimal $\tau^{y*} \geq 0$ capable of replicating the complete market solution. Intuitively, when the consumption-smoothing motive is strong, the agent has less incentive to default, and thus can get a relatively generous borrowing limit from the creditors. As the government increases the education subsidy, the individual's borrowing limit falls. But since the agent's initial borrowing limit is abundant, a fairly large education subsidy is required to drive the borrowing limit down to zero. It is possible that the rate at which the borrowing limit falls is less than growth rate of the education subsidy and, as such, the total resource available to the young — education subsidy plus borrowing limit — could rise before the borrowing limit reaches zero. In this case, the young can afford optimal human capital investment, x^*, and achieve consumption smoothing simultaneously.

4.5.2 Closed economy

In the following, I will use a numerical example to demonstrate the results of Proposition 19. The parametric specification used is as follows: $f(x) = 3x^{0.5}$, $u(\cdot) = \ln(\cdot)$, $R = 2$, $\beta = 0.99^{25} = 0.37$, and $(\omega^y, \omega^o) = (0, 1)$. Then we can compute $\widehat{\omega}^m = 2.31$ and $\overline{\omega}^m = 4.11$. Hence for any $\omega^m \in [2.31, 4.11)$, there exists optimal $\tau^{y*} \geq 0$. Figure 4.1 and 4.2 respectively demonstrate the borrowing limit and human capital investment in the economy with $\omega^m = 2$ and $\omega^m = 3$. In both Figures, borrowing limit is monotonically

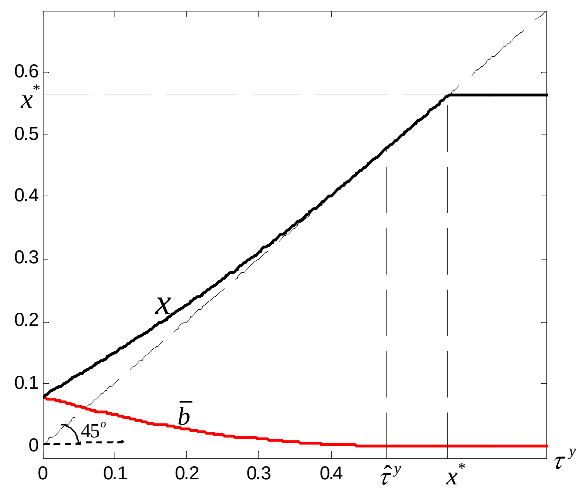


FIGURE 4.1. Non-existence of optimal $\tau^y > 0$

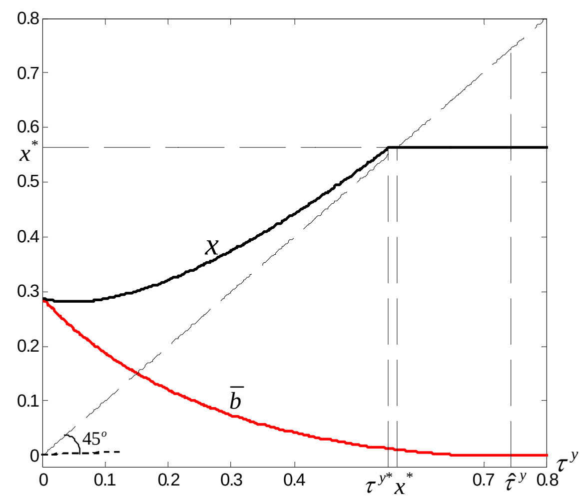


FIGURE 4.2. Existence of optimal $\tau^y > 0$

decreasing in τ^y and education investment is non-decreasing in τ^y.[8] Figure 4.1 illustrates the case that borrowing limit decrease to zero before human capital investment achieves the optimal level x^*, and therefore optimal $\tau^{y*} \geq 0$ does not exist. In contrast, Figure 4.2 exhibits that agent can obtain optimum for $\forall \tau^y \in [\tau^{y*}, \widehat{\tau}^y]$.

In the following, I examine a closed economy where the interest rate is endogenously determined instead of being exogenously given by international credit market as in the open economy. Two steps build the argument: first given complete market interest rate R^*, there exists optimal $\tau^{y*} \geq 0$; secondly given τ^{y*}, equilibrium interest rate coincides with R^*.

Consider the complete market interest rate R^*. According to Proposition 19, given R^* and appropriate values of the parameter $\omega^m \in [\widehat{\omega}^m, \overline{\omega}^m)$, there exists optimal $\tau^{y*} \geq 0$ replicating complete market solutions. Suppose $\tau^{y*} \geq 0$ optimal for that R^*, i.e. given R^*, τ^{y*} can restore complete market solution. Since x^* is the optimal level of human capital investment corresponding to R^*, young agent borrows $b^c = x^* - \tau^{y*}$. Consider her budget constraint in second period

$$c^{m,c} = \omega^m + f(x^c) - Rb^c - s^c - (1+R)\tau^{y*} \tag{4.29}$$

Since complete market solutions are replicated by τ^{y*}, $c^{m,c}$ should be equal to c^{m*}. Then by substituting $b^c = x^* - \tau^{y*}$, $x^c = x^*$ and $R = R^*$, equation (4.29) turns to

$$c^{m*} = \omega^m + f(x^*) - R^*(x^* - \tau^{y*}) - s^c - (1+R^*)\tau^{y*} \tag{4.30}$$

Moreover according to budget constraint (4.3), the complete market solution of c^{m*} is

$$c^{m*} = \omega^m + f(x^*) - R^* b^* - s^* \tag{4.31}$$

Now equating (4.30) and (4.31), and using general equilibrium condition in complete market (4.11) can yield the optimal savings of the agent at middle age

$$s^c = s^* - \tau^{y*}$$

Finally, imposing the credit market general equilibrium condition, i.e.

$$s^c = b^c \Leftrightarrow s^* - \tau^{y*} = x^* - \tau^{y*} \Leftrightarrow s^* = x^*$$

[8] For other parameter specification, education investment could be decreasing in τ^y or be decreasing first and then increasing in τ^y.

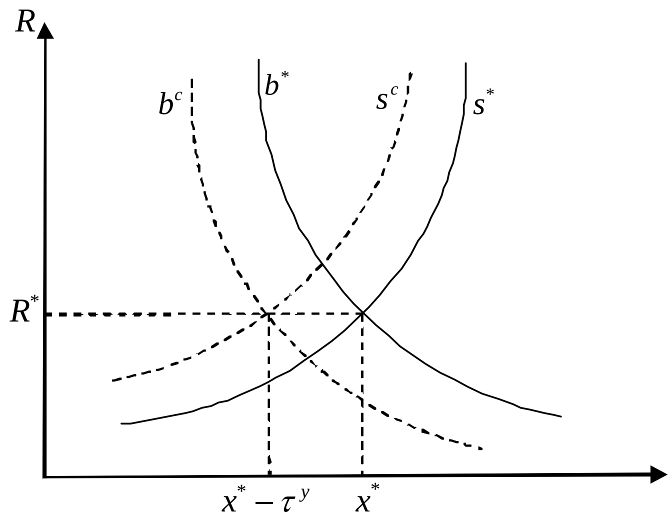


FIGURE 4.3. Credit market equilibrium

we get exactly the same equilibrium condition as complete market. Therefore the endogenous interest rate of the credit market under government intervention should be equal to R^*, and the equilibrium interest rate R^* is self-fulfilled. As illustrated in Figure 4.3, in this case both the demand curve and supply curve in the credit market move to the left at the same amount τ^{y*} and the equilibrium interest rate remains unchanged.

4.6 Conclusion

Imperfect credit market constrains the investment on human capital. Intergenerational transfers that subsidize the education of young and pension of old simultaneously is verified to be the optimal policies to replicate complete market allocation by Boldrin and Montes (2005). With respect to their study, I, following Kehoe and Levine (1993), introduce the endogenous borrowing limit in a three period OLG model with human capital investment.

The endogenous borrowing limit arises because people can not commit to repay their loan and creditors can not garnish the return of human capital. Comparing to Andolfatto and Gervais (2006) who use a numerical example to show the non-existence of optimal intergenerational policies, I derive the analytical results and demonstrate that results of Boldrin and Montes (2005) could still be valid in the setting of endogenous borrowing limit.

Consider the structure of intergenerational transfers proposed by Boldrin and Montes

(2005), the education subsidy expands borrowing limit, but in the meanwhile, tax on middle-aged agent and social pension tighten the constraints. The effect of intergenerational transfers on borrowing limit and human capital investment is not straightforward. I have shown in this paper that, if individual savings incentive is sufficiently high, there could exist optimal intergenerational transfers to replicate the complete market solutions. Finally the study of the case of closed economy, where interest rate is endogenously determined, shows given same sufficient condition there also exists an equilibrium to support optimal policy.

Chapter 5

General Conclusions

The three essays of this dissertation examine government policies against market failure. Chapter 2 evaluates the impacts of renewable energy policies on climate change. Chapter 3 explores the connection among pollution, health and growth, and studies the government policies addressing the inefficiencies arising from externality of pollution and risk sharing. Chapter 4 studies the optimal education policies countering borrowing constraints faced by young students.

From these three papers, we can learn that a careful study of policy impacts or policy design is not trivial and very important for real practice. What do we have learned so far?

First, from Chapter 2, we know that government support for renewable energy policies not necessarily alleviate the problem of climate change. If government increases subsidy for abundant backstop and the energy market is competitive, those policies definitely axcerbate the problem of climate change. After considering capacity constraints of renewable energies and recognizing the existence of market power in the fossil fuel market, the policy impacts may move to different directions, but the final policy impacts could be or be not clear and depending on model parameterization. The capacity constraints play the role because then can help renewable energy to delay the fossil fuel use to the distant future, and the existence of market power changes the optimization rule of fossil fuel owners as well as their response to renewable energy policies.

Second, Chapter 3 presents a reinforcing mechanism between pollution and growth mediated with health concern. As discussed, among all damages pollution generates, health damage is the largest and overwhelmingly dominating. If one want to learn about the interaction between economic growth and pollution, a study from the health perspective is indispensable. Chapter 3 is one of the few studies undertaking that task. It shows that an economy might experience high economic growth rate coupled with high pollution levels. This economy seems to be sustainable in terms of economic growth, but may not be desirable from a welfare standpoint. It is shown that private insurance achieves full risk sharing but does not reduce pollution; PAYG insurance reduces pollution but can only achieve partial risk sharing; pollution tax reduces pollution, but introduces an additional distortion in the rate of return to capital. In addition, we can learn from this chapter that even when double dividends of environmental tax do not exist in a static setting, they may still arise in a dynamic setting via its effects on savings behavior. More important, non-environment policies, like health insurance and pay-as-you-go social security, can help break the pollution-growth-pollution cycle. Therefore for developing countries, when developing those social institutions, they should incorporate environmental objectives in doing so.

Finally Chapter 4 shows that even with endogenous borrowing constraints, as long as individual savings incentive is sufficiently high, there could exist optimal intergenerational transfers to replicate the complete market solutions. This study saves the results of Boldrin and Montes (2005) in the framework of endogenous borrowing constraints and broadens the rationale for a two-armed (education and pension) welfare state to a much wider class of economies.

Appendix A

Additional figures and definition for Chapter 2

The following figures illustrate the impacts of renewable energy policies on residual demand and marginal revenue for the monopolist. The critical quantities of q_f in those figures are defined by the following

$$\begin{aligned} q_{f,1} &= h^{-1}(c_{b,h}) - \overline{q}_{b,l} \\ q_{f,2} &= h^{-1}(c_{b,h}) - \overline{q}_{b} \\ q_{f,3} &= h^{-1}(c_{b,s}) - \overline{q}_{b} \end{aligned}$$

$q'_{f,i}$, $i = \{1, 2, 3\}$, is the corresponding quantity after policy implementation. Definition of MR_i, $i = \{1, 2, 3\}$ is the same as before. $MR(0)$ is the marginal revenue at the initial period. $p(0)$ is the energy price at the initial period.

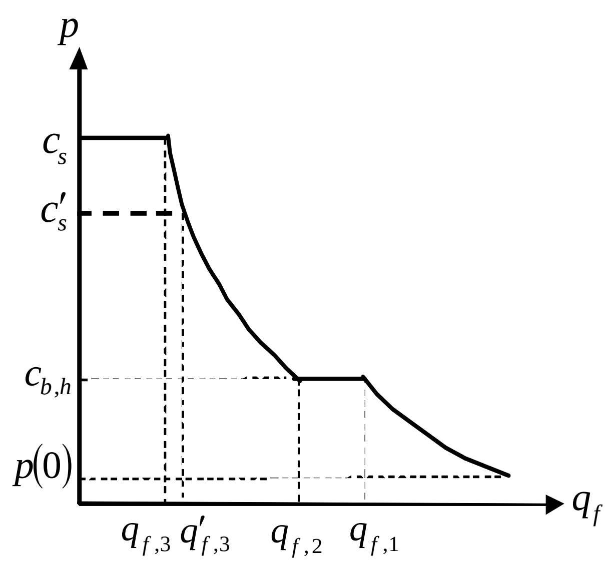


Figure A.1. Solar cost reduction policies: impact on residual demand

We define the fossil fuel stocks that gaurantee $c_{b,l} < p(0) < c_{b,h}$ in the following. First we know that there should exist two critical stock sizes of fossil fuels $X_{0,l}$ and $X_{0,h}$, with

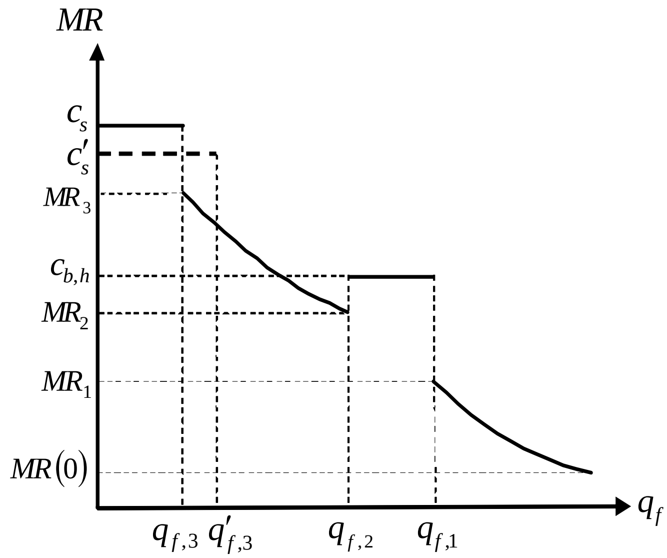


FIGURE A.2. Solar cost reduction policies: impact on marginal revenue

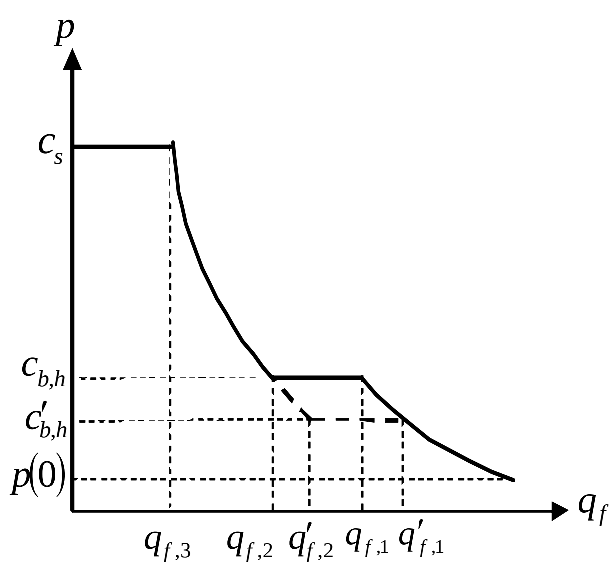


FIGURE A.3. Cost reduction policies for high cost biofuels: impact on residual demand

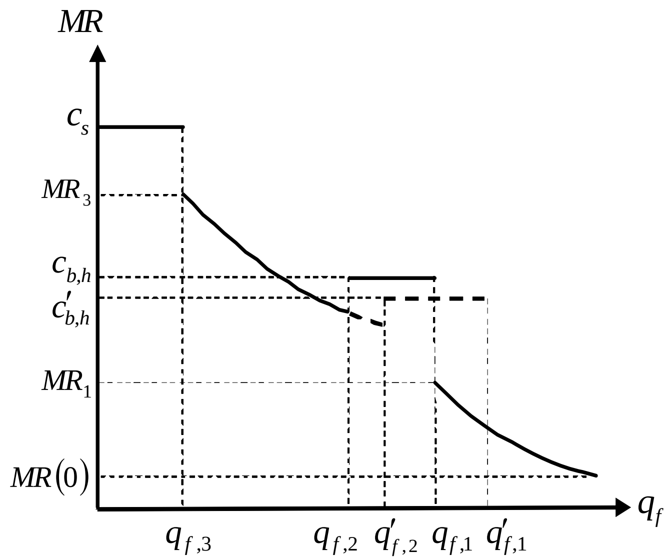


FIGURE A.4. Cost reduction policies for high cost biofuels: impact on marginal revenue

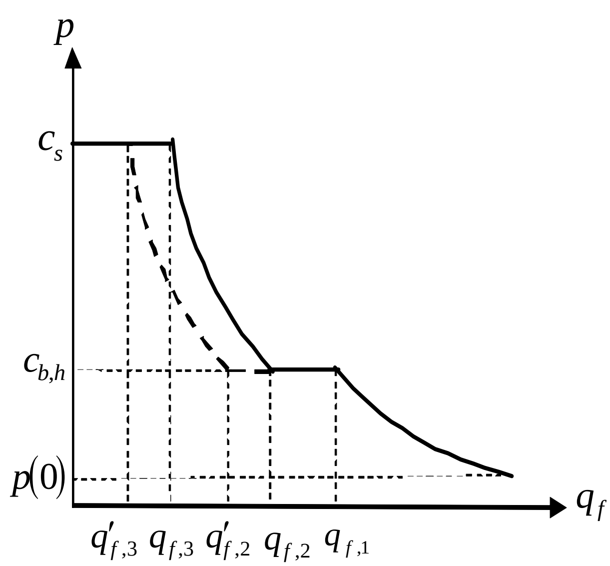


FIGURE A.5. Capacity expansion policies for high cost biofuels: impact on residual demand

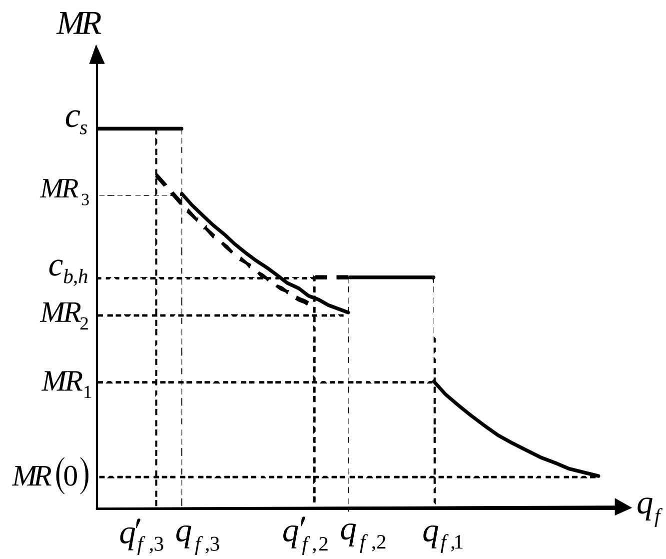


FIGURE A.6. Capacity expansion policies for high cost biofuels: impact on marginal revenue

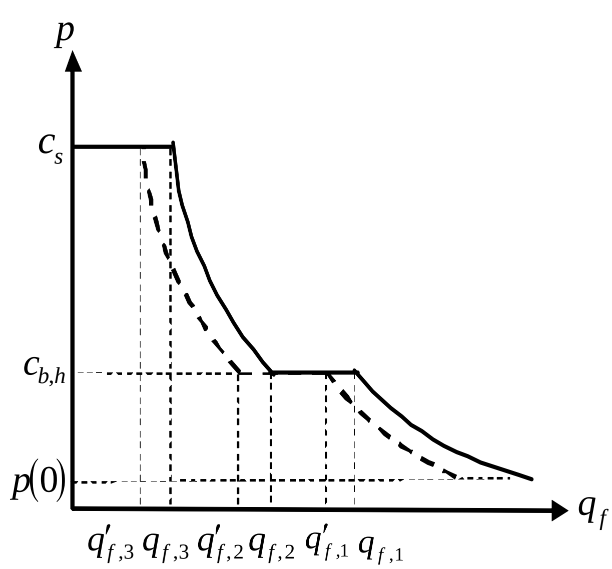


FIGURE A.7. Capacity expansion policies for low cost biofuels: impact on residual demand

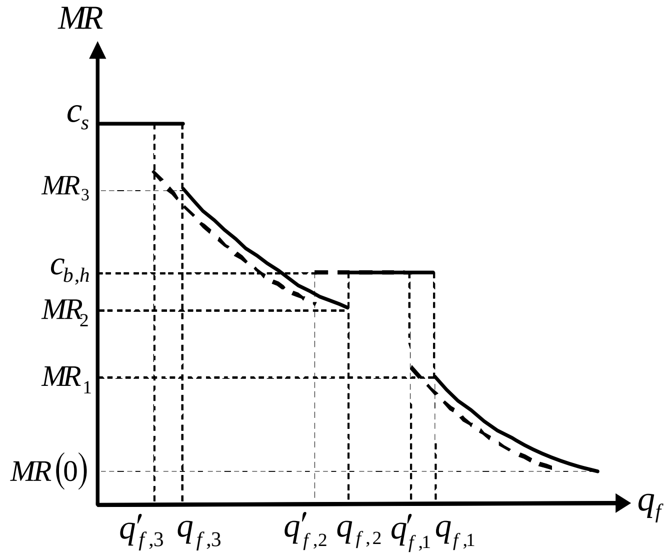


FIGURE A.8. Capacity expansion policies for low cost biofuels: impact on marginal revenue

$X_{0,l} < X_{0,h}$, such that for any $X_0 \in [X_{0,l}, X_{0,h}]$, that condition holds. $X_{0,l}$ and $X_{0,h}$ are different for the case of competitive fossil fuel market and that of non-competitive fossil fuel market.

The case of competitive fossil fuel market. By setting $T_1 = 0$ in (2.6), (2.7) and (2.8), we can define $X_{0,l}$

$$X_{0,l} = \int_0^y h^{-1}\left(c_f + (c_{b,h} - c_f)\, e^{rt}\right) dt - \overline{q}_b y \tag{A.1}$$

where y is defined by $c_f + (c_{b,h} - c_f)\, e^{ry} = c_s$. Note that if $X_0 = X_{0,l}$, current shadow value of fossil fuels equals to $c_{b,h} - c_f$ and $p(0) = c_{b,h}$. Similarly

$$X_{0,h} = \int_0^y h^{-1}\left(c_f + (c_{b,l} - c_f)\, e^{rt}\right) dt - \overline{q}_{b,l} y - \overline{q}_{b,h}(y - z) \tag{A.2}$$

where y and z are defined by $c_f + (c_{b,l} - c_f)\, e^{ry} = c_s$ and $c_f + (c_{b,l} - c_f)\, e^{rz} = c_{b,h}$.

The case of non-competitive fossil fuel market. We define $X_{0,l}$ by

$$X_{0,l} = X(T_3) + \left[h^{-1}(c_{b,h}) - \overline{q}_{b,l}\right] y + \int_y^z q_f(t)\, dt \tag{A.3}$$

where, given $\mu = h'\left(h^{-1}\left(c_{b,h}\right)\right)\left[h^{-1}\left(c_{b,h}\right) - \overline{q}_b\right] + c_{b,h} - c_f$, $q_f\left(t\right)$ is determined by (2.16), z is the solution for T_3 by solving (2.23) and y is the solution for T_2 by solving (2.15) and (2.22). Similarly, the upper bound of the stock size is

$$X_{0,h} = X\left(T_3\right) + \left[h^{-1}\left(c_{b,h}\right) - \overline{q}_{b,l}\right]\left(y - x\right) + \int_0^x q_f\left(t\right) dt + \int_y^z q_f\left(t\right) dt \qquad \text{(A.4)}$$

where, given $\mu = h'\left(h^{-1}\left(c_{b,l}\right)\right)\left[h^{-1}\left(c_{b,l}\right) - \overline{q}_{b,l}\right] + c_{b,l} - c_f$, $q_f\left(t\right)$ in the first and second integration is determined by (2.13) and (2.16) respectively, x solves T_1 in (2.21), and y and z are defined in the same way as that for $X_{0,l}$.

Appendix B

Data description for Chapter 3

The household survey data are obtained from Chinese Household Income Project 2002, which is available at http://www.icpsr.umich.edu/icpsrweb/ICPSR/studies/21741?archive=ICPSR\&q=Chinese+Household+Income+Project. The survey was conducted in 70 cities in 2002. The sample includes 6835 households. In this data, 157 extremely large negative savings rates (e.g., −1050%) are dropped from the sample by the outlier test in Greene (Page 60, 2003). The average household savings rate of the dropped outliers is −112%. All income data are adjusted by the spatial price index in Brandt and Holz (2006).

The air pollution data of 39 cities in 2002 are obtained from China Environmental Yearbook 2003 and City Environmental Report 2003. The city air pollution composite index (C_Air) is published by the Ministry of Environmental Protection of China, and is equal to the summation of ratios of the annual average concentration rates of three pollutants' (SO_2, NO_2 and PM_{10}) to their respective Air Quality Class 2 standards in China. In Table B.1, H_* represents household variables, HH_* represents household head variables and C_* represents city variables. C_Rain and C_Wind have been collected from http://cdc.cma.gov.cn/shuju and City Statistical Yearbook 2003.

TABLE B.1. Description of variables

Variable	Description
H_Save	(income – consumption)/income (%)
H_Inc	per capita income (1000 CNY)
H_Inc2	H_Inc×H_Inc
H_Size	household size
H_Emp	NFM being employed
H_Child*	=1 if having child aged 1~14
H_Hcap	NFM being handicapped
H_Ill	NFM getting severe illness in 2002
H_Insur	NFM having public insurance
HH_Age	age of household head
HH_Edu	schooling years of household head
HH_Sex*	=1 if household head is male
C_Air	city air composite pollution index
C_Pop	city population (10,000)
C_Rinc	city per capital income (1000 CNY)
C_Manu	percentage of secondary industry in GDP (%)
C_Rain	annual rainfall (1000 millimeters)
C_Wind	wind speed (10 meters/second)

Note: symbol ∗ represents dummy variable. NFM represents Number of Family Member.

TABLE B.2. Statistics of variables

Variable	Min	Max	Mean	Std.
H_Save	−76.89	90.051	24.116	24.331
H_Inc	0.734	64.449	8.297	4.945
H_Inc2	0.539	4153	93.291	155.758
H_Size	1	9	3.017	0.772
H_Emp	0	4	1.502	0.809
H_Child*	0	1	0.271	0.444
H_Hcap	0	5	0.471	0.794
H_Ill	0	5	0.191	0.563
H_Insur	0	5	0.691	0.926
HH_Age	20	92	48.551	11.014
HH_Edu	0	23	10.842	3.294
HH_Sex*	0	1	0.654	0.476
C_Air	0.41	5.51	2.881	1.045
C_Pop	6.2	787.5	237.558	228.753
C_Rinc	4.961	10.713	7.472	1.409
C_Manu	22.98	64.72	45.501	9.116
C_Rain	0.242	2.677	0.891	0.525
C_Wind	0.8	5.3	2.155	0.845

Appendix C

Proofs

Proof. [Proof for Proposition 1] First, we consider the case of cost reduction of solar. Comparative statics of (2.6) — (2.8) lead to

$$\begin{pmatrix} \int_0^T \frac{e^{rt}}{h'(h^{-1}(p(t)))}dt & \overline{q}_{b,h} & h^{-1}(c_s) - \overline{q}_b \\ e^{rT_1} & r\lambda e^{rT_1} & 0 \\ e^{rT} & 0 & r\lambda e^{rT} \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial c_s} \\ \frac{\partial T_1}{\partial c_s} \\ \frac{\partial T}{\partial c_s} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Denote $\triangle_{ce}$ the determinant of the square matrix

$$\triangle_{ce} = r\lambda e^{r(T_1+T)} \left[r\lambda \int_0^T \frac{e^{rt}}{h'\left(h^{-1}\left(p\left(t\right)\right)\right)}dt - h^{-1}\left(c_s\right) + \overline{q}_{b,l} \right] < 0$$

Then applying Cramer rule, we obtain

$$\begin{aligned} \frac{\partial \lambda}{\partial c_s} &= -r\lambda e^{rT_1}\left[h^{-1}\left(c_s\right) - \overline{q}_b\right]/\triangle_{ce} > 0 \\ \frac{\partial T_1}{\partial c_s} &= e^{rT_1}\left[h^{-1}\left(c_s\right) - \overline{q}_b\right]/\triangle_{ce} < 0 \\ \frac{\partial T}{\partial c_s} &= \left[r\lambda e^{rT_1} \int_0^T \frac{e^{rt}}{h'\left(h^{-1}\left(p\left(t\right)\right)\right)}dt - \overline{q}_{b,h} e^{rT_1}\right]/\triangle_{ce} > 0 \end{aligned}$$

Similarly, we can obtain the effects of cost reduction policies for high cost biofuels,

$$\begin{aligned} \frac{\partial \lambda}{\partial c_{b,h}} &= -\overline{q}_{b,h} r\lambda e^{rT}/\triangle_{ce} > 0 \\ \frac{\partial T_1}{\partial c_{b,h}} &= \left\{ r\lambda e^{rT} \int_0^T \frac{e^{rt}}{h'\left(h^{-1}\left(p\left(t\right)\right)\right)}dt - e^{rT}\left[h^{-1}\left(c_s\right) - \overline{q}_b\right] \right\}/\triangle_{ce} > 0 \\ \frac{\partial T}{\partial c_{b,h}} &= \overline{q}_{b,h} e^{rT}/\triangle_{ce} < 0 \end{aligned}$$

effects of capacity expansion policies for high cost biofuels,

$$\begin{aligned}
\frac{\partial \lambda}{\partial \overline{q}_{b,h}} &= (r\lambda)^2 e^{r(T_1+T)} (T - T_1) / \triangle_{ce} < 0 \\
\frac{\partial T_1}{\partial \overline{q}_{b,h}} &= -(T - T_1) r\lambda e^{r(T_1+T)} / \triangle_{ce} > 0 \\
\frac{\partial T}{\partial \overline{q}_{b,h}} &= -(T - T_1) r\lambda e^{r(T_1+T)} / \triangle_{ce} > 0
\end{aligned}$$

and effects of capacity expansion policies for low cost biofuels,

$$\begin{aligned}
\frac{\partial \lambda}{\partial \overline{q}_{b,l}} &= (r\lambda)^2 e^{r(T_1+T)} T / \triangle_{ce} < 0 \\
\frac{\partial T_1}{\partial \overline{q}_{b,l}} &= -Tr\lambda e^{r(T_1+T)} / \triangle_{ce} > 0 \\
\frac{\partial T}{\partial \overline{q}_{b,l}} &= -Tr\lambda e^{r(T_1+T)} / \triangle_{ce} > 0
\end{aligned}$$

□

Proof. [Proof for Corollary 2] To prove the corollary, we only need to characterize the sign of $\partial q_f(t) / \partial \overline{q}_{b,l}$ at $t = 0$. First, we can derive the policy impacts on fossil fuel use by applying implicit theorem in (2.4) and (2.5)

$$\frac{\partial q_f(t)}{\partial \overline{q}_{b,l}} = \frac{e^{rt} \frac{\partial \lambda}{\partial \overline{q}_{b,l}}}{h'\left(h^{-1}\left(\lambda e^{rt} + c_f\right)\right)} - 1$$

By substituting $\partial \lambda / \partial \overline{q}_{b,l}$, we have, for $t \in [0, T]$

$$\frac{\partial q_f(t)}{\partial \overline{q}_{b,l}} = \frac{\frac{e^{rt} r\lambda}{h'\left(h^{-1}\left(\lambda e^{rt} + c_f\right)\right)} T}{\int_0^T \frac{e^{rt} r\lambda}{h'\left(h^{-1}\left(\lambda e^{rt} + c_f\right)\right)} dt - h^{-1}(c_s) + \overline{q}_{b,l}} - 1$$

Note that the denominator of the first item is negative and constant. The numerator of the first item is negative and decreasing in time t for all linear demand functions. Suppose $\partial q_f(t) / \partial \overline{q}_{b,l} \geq 0$ at $t = 0$, then $\partial q_f(t) / \partial \overline{q}_{b,l} \geq 0$ for all $t \in [0, T]$. That means fossil fuel extraction increases for all periods. But according to Proposition 1, we also have $\partial T / \partial \overline{q}_{b,l} > 0$, which implies that resource exhaustion condition is violated. Therefore we must have $\partial q_f(t) / \partial \overline{q}_{b,l} < 0$ at $t = 0$. □

Proof. [Proof for Proposition 4] First we can rewrite monopolist's optimization problem as

$$Max \int_0^T e^{-rt} H(t)\, dt,$$

Then optimal T_2 can be solved by the problem that given optimal solutions of $q_f(t)$, $q_{b,l}(t)$, $q_{b,h}(t)$, $q_s(t)$, μ, T_1, T_3, and T,

$$\underset{T_2}{Max} \int_0^{T_1} e^{-rt} H(t)\, dt + \int_{T_1}^{T_2} e^{-rt} H(t)\, dt + \int_{T_2}^{T_3} e^{-rt} H(t)\, dt + \int_{T_3}^{T} e^{-rt} H(t)\, dt$$

For $t \in [T_1, T_2]$, $q_f(t)$ has corner solution and

$$H(t) = \left[h^{-1}(c_{b,h}) - \overline{q}_{b,l}\right]\left(c_{b,h} - c_f - \mu e^{rt}\right).$$

For $t \in [T_2, T_3]$, $q_f(t)$ has interior solution and

$$H(t) = q_f(t)\left[h\left(q_f(t) + \overline{q}_b\right) - c_f - \mu e^{rt}\right].$$

The results follows directly by optimizing the objective function with respect to T_2. □

Proof. [Proof for Proposition 6] The proof is based on comparative statics study of (2.12), (2.15) and (2.20) $\sim$ (2.23). (2.20) $\sim$ (2.23) and (2.15) are five equations with five unknowns. We can derive policy effects on $\{\mu, T_1, T_2, T_3, X(T_3)\}$ first and then use (2.12) to obtain policy effect on T. Applying the implicit function theorem to equations (2.20) $\sim$ (2.23) and (2.15), we have

$$\Omega \times \frac{\partial w}{\partial c_s} = \begin{pmatrix} -\frac{\partial X(T_3)}{\partial c_s} \\ 0 \\ 0 \\ -\frac{\partial MR(T_3)}{\partial c_s} \\ 0 \end{pmatrix} \tag{C.1}$$

where

$$\Omega = \begin{pmatrix} A & 0 & B & h^{-1}(c_s) - \overline{q}_b & 1 \\ -e^{rT_1} & -r\mu e^{rT_1} & 0 & 0 & 0 \\ -e^{rT_2} & 0 & -r\mu e^{rT_2} & 0 & \frac{\partial MR(T_2)}{\partial q_f(T_2)} \\ -e^{rT_3} & 0 & 0 & -r\mu e^{rT_3} & 0 \\ e^{rT_2}B & 0 & r\mu e^{rT_2}B & 0 & 0 \end{pmatrix}$$

$$w = (\mu, T_1, T_2, T_3, q_f(T_2))'$$

and

$$A = \int_0^{T_1} \frac{e^{rt}}{\partial MR(t)/\partial q_f(t)} dt + \int_{T_2}^{T_3} \frac{e^{rt}}{\partial MR(t)/\partial q_f(t)} dt < 0$$

$$B = h^{-1}(c_{b,h}) - \overline{q}_{b,l} - q_f(T_2)$$

$$\frac{\partial MR(T_2)}{\partial q_f(T_2)} = h''(q_f(T_2) + \overline{q}_b) q_f(T_2) + 2h'(q_f(T_2) + \overline{q}_b) < 0$$

$$\frac{\partial X(T_3)}{\partial c_s} = \frac{\ln\left(\frac{c_s - c_f}{MR(T_3)}\right) + h'(h^{-1}(c_s))(h^{-1}(c_s) - \overline{q}_b)\left[\frac{1}{c_s - c_f} - \frac{\frac{\partial MR(T_3)}{\partial c_s}}{MR(T_3)}\right]}{rh'(h^{-1}(c_s))}$$

$$\frac{\partial MR(T_3)}{\partial c_s} = \frac{(h^{-1}(c_s) - \overline{q}_b) h''(h^{-1}(c_s))}{h'(h^{-1}(c_s))} + 2 > 0$$

Item A is derived by differentiating (2.20) with respect to μ, in which $\partial q_f(t)/\partial\mu$ is obtained by applying implicit function theorem in (2.13) and (2.16). Since revenue function of monopolist is concave, it is easy to check signs of A, $\partial MR(T_3)/\partial c_s$ and $\partial MR(T_2)/\partial q_f(T_2)$. Denote the determinant of the square matrix Ω as $\det(\Omega)$. We have

$$\det(\Omega) = Br^2\mu^2 e^{r(T_1+T_2+T_3)}\left[h^{-1}(c_s) - \overline{q}_b + B - r\mu A\right]\frac{\partial MR(T_2)}{\partial q_f(T_2)} < 0$$

Then applying Cramer rule in (C.1), we obtain

$$\frac{\partial\mu}{\partial c_s} = Br^2\mu^2 e^{r(T_1+T_2)} \frac{\frac{\partial MR(T_2)}{\partial q_f(T_2)}\left[r\mu e^{rT_3}\frac{\partial X(T_3)}{\partial c_s} + (h^{-1}(c_s) - \overline{q}_b)\frac{\partial MR(T_3)}{\partial c_s}\right]}{\det(\Omega)}$$

$$\frac{\partial T_1}{\partial c_s} = \frac{\partial T_2}{\partial c_s} = \frac{\partial T}{\partial c_s} = -\frac{\partial\mu}{\partial c_s}/(r\mu)$$

$$\frac{\partial q_f(T_2)}{\partial c_s} = 0$$

According to first two equations, signs of $\partial\mu/\partial c_s$ and $\partial T_i/\partial c_s$, $i = \{1, 2\}$ depend on sign of the item in the square bracket, which, by substituting $\partial X(T_3)/\partial c_s$, $\partial MR(T_3)/\partial c_s$ and (2.23), can be simplified to

$$\left(h^{-1}(c_s) - \overline{q}_b\right)\left[-(1+\theta)\ln\left(1+\frac{1}{\theta}\right) + 1 + \frac{1}{\theta}\right]$$

where $\theta = (c_s - c_f)/\left[\left(h^{-1}(c_s) - \overline{q}_b\right)h'\left(h^{-1}(c_s)\right)\right]$. Therefore $\partial\mu/\partial c_s < 0$ and $\partial T_i/\partial c_s > 0$, $i = \{1, 2\}$ if and only if

$$F(\theta) = -(1+\theta)\ln\left(1+\frac{1}{\theta}\right) + 1 + \frac{1}{\theta} < 0$$

Note $\theta < -1$. we derive the sign of $F(\theta)$ by characterizing its curvature. Firstly, we have

$$\begin{aligned} F'(\theta) &= -\ln\left(1+\frac{1}{\theta}\right) + \frac{1}{\theta} - \frac{1}{\theta^2} \\ F''(\theta) &= \frac{1}{\theta(\theta+1)} + \frac{2-\theta}{\theta^3} \end{aligned}$$

Then from equation of $F''(\theta)$, we can learn $F''(\theta) > 0$ if and only if $\theta \in (-2, -1)$. Moreover, $F'(\theta^*) = 0$ has unique solution $\theta^* = -1.4624$, which is in the convex range of $F(\theta)$ and thus minimizes $F(\theta)$. Therefore, $F(\theta)$ decreases in $\theta \in (-\infty, \theta^*)$ first and then increases in $\theta \in (\theta^*, -1)$. Now if we can show $\lim_{\theta\to-\infty} F(\theta) \le 0$ and $\lim_{\theta\to--1} F(\theta) \le 0$, we have $F(\theta) < 0$ for all $\theta < -1$. By L'Hoptial rule

$$\lim_{\theta\to-\infty} F(\theta) = \lim_{\theta\to-\infty}\left[-\frac{\ln\left(1+\frac{1}{\theta}\right)}{\frac{1}{1+\theta}} + 1 + \frac{1}{\theta}\right] = \lim_{\theta\to-\infty}\left[-\frac{\frac{1}{\theta(\theta+1)}}{\frac{1}{(1+\theta)^2}} + 1 + \frac{1}{\theta}\right] = 0$$

Similarly, $\lim_{\theta\to-1} F(\theta) = 0$. Therefore, $F(\theta) < 0$ for all $\theta \in (-\infty, -1)$. That proves $\partial\mu/\partial c_s < 0$ and $\partial T_i/\partial c_s > 0$, $i = \{1, 2\}$.

$\partial T_3/\partial c_s > 0$ is obtained by the fact that the new price path after policy implementation between T_2 and T_3 can not cross the original one. If the two price paths cross with each other at some time $\widetilde{t} \in [T_2, T_3]$, then the marginal revenue of monopolist at $\widetilde{t}$ is equalized before and after policy implementation. Moreover, optimum requires equality between marginal revenue and augmented marginal cost for all $t \in [T_2, T_3]$. That implies at time $\widetilde{t}$, rent value of fossil fuels before policy implementation are the same as that after policy implementation, which contradicts with the fact that $\partial\mu/\partial c_s < 0$.

Finally, $\partial T/\partial c_s > 0$ can be easily derived from (2.12) and $\partial\mu/\partial c_s < 0$. □

Proof. [Proof for Propositioin 7] Following the same exercise of comparative statics in Proposition 6, we have

$$\Omega \times \frac{\partial w}{\partial c_{b,h}} = \begin{pmatrix} -\frac{T_2 - T_1}{h'\left(h^{-1}\left(c_{b,h}\right)\right)} \\ -\frac{\partial MR(T_1)}{\partial c_{b,h}} \\ 0 \\ 0 \\ -\frac{\mu\left(e^{rT_2} - e^{rT_1}\right)}{h'\left(h^{-1}\left(c_{b,h}\right)\right)} \end{pmatrix} \tag{C.2}$$

where

$$\frac{\partial MR\left(T_1\right)}{\partial c_{b,h}} = h''\left(h^{-1}\left(c_{b,h}\right)\right)\frac{h^{-1}\left(c_{b,h}\right) - \overline{q}_{b,l}}{h'\left(h^{-1}\left(c_{b,h}\right)\right)} + 2 > 0$$

Then by applying Cramer rule, we have

$$\begin{aligned}
\frac{\partial \mu}{\partial c_{b,h}} &= -r^3\mu^3 e^{r(T_1+T_2+T_3)}\frac{\mu\left(e^{rT_2} - e^{rT_1}\right) + B\frac{\partial MR(T_2)}{\partial q_f(T_2)}\left[\left(1 - e^{r(T_1-T_2)}\right)/r + T_1 - T_2\right]}{h'\left(h^{-1}\left(c_{b,h}\right)\right) \times \det\left(\Omega\right)} \\
\frac{\partial T_2}{\partial c_{b,h}} &= -r^2\mu^2 e^{r(T_1+T_2+T_3)}\frac{\frac{\partial MR(T_2)}{\partial q_f(T_2)}\left[\begin{array}{c} \frac{\left(1-e^{r(T_1-T_2)}\right)\left(h^{-1}(c_s)-\overline{q}_b - rA\mu\right)}{r} \\ +B\left(T_2 - T_1\right)\end{array}\right] - \mu\left(e^{rT_2} - e^{rT_1}\right)}{h'\left(h^{-1}\left(c_{b,h}\right)\right) \times \det\left(\Omega\right)} \\
\frac{\partial T_3}{\partial c_{b,h}} &= \frac{\partial T}{\partial c_{b,h}} = -\frac{\partial \mu}{\partial c_{b,h}}/\left(r\mu\right) \\
\frac{\partial q_f\left(T_2\right)}{\partial c_{b,h}} &= -r^2\mu^2 e^{r(T_1+T_2+T_3)}\frac{\mu\left(e^{rT_2} - e^{rT_1}\right)}{h'\left(h^{-1}\left(c_{b,h}\right)\right)}\left[h^{-1}\left(c_s\right) - \overline{q}_b + B - r\mu A\right]/\det\left(\Omega\right)
\end{aligned}$$

It is easy to check that $\partial T_2/\partial c_{b,h} > 0$ and $\partial q_f\left(T_2\right)/\partial c_{b,h} < 0$, and it is remained to verify $\partial\mu/\partial c_{b,h} < 0$. Denote $G = 1 - e^{r(T_1-T_2)} + r\left(T_1 - T_2\right)$. To show $\partial\mu/\partial c_{b,h} < 0$, it is enough to show $G < 0$. By some arrangement, we have

$$G\left(z\right) = 1 - z + \ln z$$

where $z = e^{r(T_1-T_2)}$. Since $T_1 < T_2$, $0 < z < 1$. Moreover, we have $\lim_{z\to 0} G\left(z\right) = -\infty$, $\lim_{z\to 1} G\left(z\right) = 0$ and $G'\left(z\right) = 1/z - 1 > 0$. Therefore $G\left(z\right) > 0$ for all $z \in (0, 1)$ and thus $\partial\mu/\partial c_{b,h} < 0$ is proved. $\partial T_1/\partial c_{b,h} > 0$ follows the similar argument in proof for Proposition 6 that, between 0 and T_1, the new price path after policy implementation can not cross the one before policy implementation. □

Proof. [Proof of Proposition 9] We prove the existence of non-trivial stable steady state by examining the characteristics of phase diagrams. Denote $\triangle k_t = k_{t+1} - k_t$ and $\triangle P_t =$

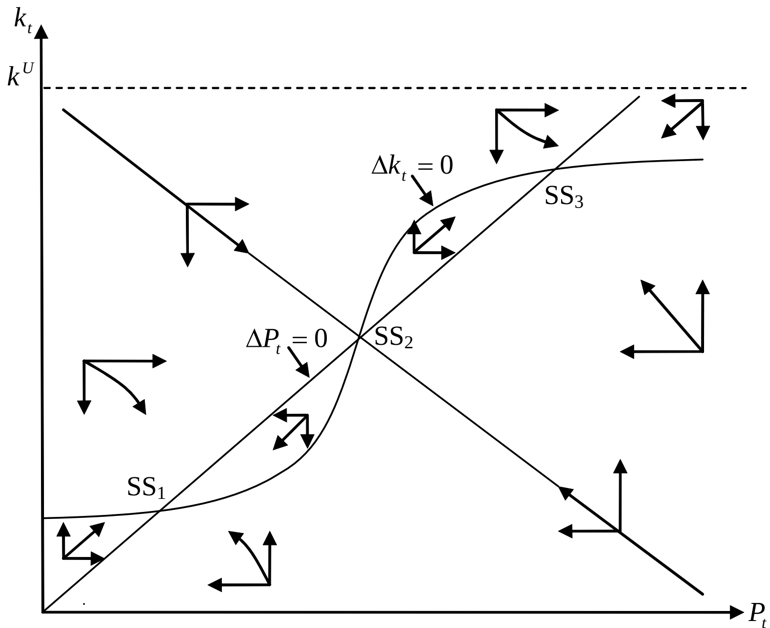


FIGURE C.1. Dynamics of pollution and capital with multiple steady states

$P_{t+1} - P_t$, phaselines are defined as the follows:

1. $\triangle P_t = 0$ implies that k_t is linear in P_t with $k_t = \zeta P_t/\rho_t$. For $k_t > \zeta P_t/\rho$, $\triangle P_t > 0$.

2. $\triangle k_t = 0$ implies that the phaseline is either $k_t = 0$ or $k_t = [\theta_t(1-\alpha)A]^{\frac{1}{1-\alpha}}$. For the latter one, it is easy to check that it is strictly positive at $P_t = 0$ and increasing in P_t.[1] In addition, as P_t increases to infinite, σ converges to 1 and consequently k_t converges to its up bound $\overline{k}^U = [(\omega\psi\delta + \delta)(1-\alpha)A/(\omega\psi\delta + \delta + 1)]^{\frac{1}{1-\alpha}}$. Moreover, as P_t increases to infinite, the slope of that pahseline converges to zero , i.e. $dk_t/dP_t|_{P_t \longrightarrow \infty} = 0$. For $k_t > [\theta_t(1-\alpha)A]^{\frac{1}{1-\alpha}}$, $\triangle k_t < 0$.

Examples of the two phaselines are illustrated in Figure 3.2 and C.1. Consider the two phaselines $k_t = 0$ and $k_t = \zeta P_t/\rho_t$, $(0,0)$ is a steady state. For the other phaseline of $\triangle k_t = 0$, since it is above and below $\triangle P_t = 0$ at $P_t = 0$ and $P_t = \infty$ respectively, it has to intersect $\triangle P_t = 0$ from above at least once. In addition, according to the figures, the corresponding steady state is a sink and saddle as $\triangle k_t = 0$ intersects $\triangle P_t = 0$ from above and from below respectively. Therefore, there must exist a non-trivial stable steady state, and if the non-trivial steady state is unique, it must be a sink.

In the following, we derive the sufficient condition of uniqueness by checking the

[1] $\frac{dk_t}{dP_t} = [(1-\alpha)A]^{\frac{1}{1-\alpha}} \frac{1}{1-\alpha}\theta_t^{\frac{\alpha}{1-\alpha}} \frac{(1-\zeta)\omega\psi\delta\sigma\prime(P_{t+1})}{[\omega\psi\delta\sigma(P_{t+1})+\delta+1]^2} > 0$

steady state solutions. The non-trivial steady state solution $\overline{k}$ is solved by (3.15). Define the right hand side of (3.15) as a function

$$M(x) = \frac{\omega\psi\delta\sigma(\frac{\rho x}{\zeta}) + \delta}{\omega\psi\delta\sigma(\frac{\rho x}{\zeta}) + \delta + 1}(1-\alpha)Ax^{\alpha}$$

Since left hand side of (3.15) is 45^o line, $M(0) = 0$ and $dM/dx\,|_{x=0} = \infty$, if $M(x)$ crosses the 45^o line more than once, i.e. non-trivial $\overline{k}$ is not unique, it must cross from below at least once. Therefore, if the slop of $M(x)$ evaluated at $x = \overline{k}$ is always less than one, i.e. always cross from above, the non-trivial steady state solution must be unique. Therefore the uniqueness requires

$$\frac{dM}{dx}\,|_{x=\overline{k}} = \frac{\omega\psi\delta\sigma'(\frac{\rho\overline{k}}{\zeta})\frac{\rho\overline{k}}{\zeta}}{\left[\omega\psi\delta\sigma(\frac{\rho\overline{k}}{\zeta}) + \delta + 1\right]\left[\omega\psi\delta\sigma(\frac{\rho\overline{k}}{\zeta}) + \delta\right]} + \alpha < 1$$

which is equivalent to

$$\frac{\sigma'(\overline{P})\frac{\rho\overline{k}}{\zeta}}{\sigma(\overline{P})} < \left[\frac{\delta+1}{\omega\psi\sigma(\overline{P})} + \omega\psi\delta\sigma(\overline{P}) + \delta + 2\right](1-\alpha)$$

Since $\sigma\left(\overline{P}\right)$ is concave, $\sigma'(\overline{P})\rho\overline{k}/\zeta \leq \sigma(\overline{P})$ holds. Moreover by setting σ equal to zero or one, right hand side of the above inequality is larger than $[(\delta+1)/(\omega\psi) + \delta + 2](1-\alpha)$. Therefore, $[(\delta+1)/(\omega\psi) + \delta + 2](1-\alpha) > 1$ is sufficient for the non-trivial steady state to be unique and asymptotically stable. □

Proof. [Proof of Proposition 10] The proof follows directly from Proposition 1. Applying the implicit function theorem to (3.15), we have

$$\frac{\partial\overline{k}}{\partial\rho} = -\frac{\frac{\omega\psi\delta\sigma'(\frac{\rho\overline{k}}{\zeta})\frac{\overline{k}}{\zeta}}{\left[\omega\psi\delta\sigma(\frac{\rho\overline{k}}{\zeta})+\delta+1\right]^2}(1-\alpha)A\overline{k}^{\alpha}}{\frac{\omega\psi\delta\sigma'(\frac{\rho\overline{k}}{\zeta})\frac{\rho\overline{k}}{\zeta}}{\left[\omega\psi\delta\sigma(\frac{\rho\overline{k}}{\zeta})+\delta+1\right]\left[\omega\psi\delta\sigma(\frac{\rho\overline{k}}{\zeta})+\delta\right]} + \alpha - 1} \tag{C.3}$$

According to the sufficient condition of uniqueness in Proposition 1, the denominator of (C.3) is strictly less than zero. Hence, $\partial\overline{k}/\partial\rho > 0$. The proof applies for $\partial\overline{k}/\partial\zeta < 0$. The effects on $\overline{P}$ follows immediately from (3.16). □

Proof. [Proof of Proposition 11] The proof follows by differentiate the following steady state utility

$$\overline{U} = \ln\left(\overline{w} - \overline{s}\right) + \delta\left\{\sigma\left(\cdot\right)\left[\omega \ln\left(\psi \overline{m}^{\psi}\right) + \ln\left(\overline{s}\overline{R} - \overline{m}\right)\right] + \left[1 - \sigma\left(\cdot\right)\right]\ln\left(\overline{s}\overline{R}\right)\right\}$$

with respect to ρ

$$\frac{\partial \overline{U}}{\partial \rho} = \frac{\partial \overline{U}}{\partial \overline{s}}\frac{\partial \overline{s}}{\partial \rho} + \frac{\partial \overline{U}}{\partial \overline{m}}\frac{\partial \overline{m}}{\partial \rho} + \left(\frac{\partial \overline{U}}{\partial \overline{w}}\frac{\partial \overline{w}}{\partial \overline{k}} + \frac{\partial \overline{U}}{\partial \overline{R}}\frac{\partial \overline{R}}{\partial \overline{k}}\right)\frac{\partial \overline{k}}{\partial \rho} + \frac{\partial \overline{U}}{\partial \overline{P}}\frac{\partial \overline{P}}{\partial \rho}$$

By substituting first order conditions $\partial \overline{U}/\partial \overline{s} = \partial \overline{U}/\partial \overline{m} = 0$ and equilibrium conditions (3.3) (3.2) (3.11) (3.15), $\partial \overline{U}/\partial \rho$ can be simplified to

$$\frac{\partial \overline{U}}{\partial \rho} = \underbrace{\left(\overline{R} - 1\right)\frac{1 - \alpha}{\left(1 - \alpha\right)A\overline{k}^{\alpha} - \overline{k}}\frac{\partial \overline{k}}{\partial \rho}}_{\text{capital effect}} + \underbrace{\left(\frac{\rho}{\zeta}\frac{\partial \overline{k}}{\partial \rho} + \frac{\overline{k}}{\zeta}\right)\frac{\partial \overline{U}}{\partial \overline{P}}}_{\text{health effect}} \tag{C.4}$$

Since $\partial \overline{k}/\partial \rho > 0$ and $\partial \overline{U}/\partial \overline{P} < 0$, we have

$$\frac{\partial \overline{U}}{\partial \rho} < 0 \quad \text{if } \overline{R} \leq 1$$

$R = 1$ defines the steady state capital $\widehat{k}$ where $R = f'\left(\widehat{k}\right) = 1$. Hence the steady-state utility of an economy with $\overline{k} \geq \widehat{k}$, is monotonically decreasing in ρ. □

Proof. [Proof of Proposition 12] The Lagrangian for (3.17) is given by:

$$\pounds = U_0 + \sum_{t=1}^{\infty}\beta^t \left\{\begin{array}{c} U_t + \beta\mu_{t+1}\left\{P_{t+1} - (1-\zeta)P_t - \rho k_t + G(q_t)\right\} \\ +\beta\lambda_{t+1}\left\{\begin{array}{c} f\left(k_t\right) - c_t^y - \sigma\left(P_t\right)\left(c_t^{o,d} + m_t\right) \\ -\left[1 - \sigma\left(P_t\right)\right]c_t^o - k_{t+1} - q_t \end{array}\right\}\end{array}\right\} \tag{C.5}$$

where $\beta\mu_{t+1}$ and $\beta\lambda_{t+1}$ respectively denote the Lagrangian multipliers of pollution dynamics (3.4) and resource constraint (3.18). Notice that λ_{t+1} represents the current shadow value of the capital stock (k_{t+1}) whereas μ_{t+1} represents the current shadow cost of the pollution stock (P_{t+1}).

Differentiating the Lagrangian with respect to c_t^y and P_{t+1}, we obtain the discounted shadow price of k_{t+1} as well as the shadow cost of pollution:

$$\beta\lambda_{t+1} = \frac{1}{c_t^y} \tag{C.6}$$

$$\beta\mu_{t+1} = \sum_{i=t}^{\infty} [\beta(1-\zeta)]^{i-t}\,\Omega_{i+1} \tag{C.7}$$

Substituting (C.6) and (C.7) into the first-order conditions of other variables, we can explicitly characterize the optimality conditions as (3.21) and

$$\frac{c_t^o}{\delta c_t^y} = \frac{1}{\beta} \tag{C.8}$$

$$\frac{c_{t+1}^o}{\delta c_t^y} = f'(k_{t+1}) - c_{t+1}^y \rho D_{t+2} \tag{C.9}$$

$$c_t^{o,d} = c_t^o = \frac{m_t}{\psi\omega} \tag{C.10}$$

$$\lim_{t\to\infty} \beta^t \lambda_t k_t = 0 \tag{C.11}$$

where (C.11) is the transversality condition. Given initial values of k_0 and P_{-1}, the optimal solution is an infinite sequence $\left\{k_t, m_t, q_t, P_t, c_t^y, c_t^{o,d}, c_t^o\right\}_{t=0}^{\infty}$ satisfying (3.21), (C.8) – (C.11), and constraints (3.18) and (3.4).

Equation (C.8) represents the optimal consumption allocation between generations; it equates the intergenerational marginal rate of substitution with the planner's subjective discount rate. Equation (C.9) shows the optimal allocation of *intra*generational consumption. The marginal rate of substitution between present consumption and future consumption is equal to the net marginal value of capital. Since one unit capital in period $t+1$ generates ρ units of pollution in period $t+2$ (cf. (3.4)), ρD_{t+2} is the marginal damage of k_{t+1} measured in utils. Dividing it by the marginal utility of consumption, $1/c_{t+1}^y$, $c_{t+1}^y \rho D_{t+2}$ measures the marginal damage evaluated in monetary units.

From (C.8) and (C.9), we obtain $f'(k_{t+1}) = c_{t+1}^y / (\beta c_t^y) + c_{t+1}^y \rho D_{t+2}$, which implies that, at the steady state, $f'(\overline{k}) = 1/\beta + \overline{c}^y \rho \overline{D}$. Thus, $\overline{k}$ is decreasing in ρ and $\overline{D}$. Without pollution, i.e., when $\rho = 0$, $f'(\overline{k}) = 1/\beta$, which is the standard modified golden rule in the Diamond (1965) model. It follows that, upon internalizing the externality of pollution, $f'(\overline{k}) > 1/\beta$, implying the socially-optimal capital stock is lower than the capital stock associated with modified golden rule.

Equation (C.10) shows the optimal allocation of consumption between the old in poor health status and the old in good health status, and the optimal allocation between consumption and health expenditure. The planner is able to provide complete risk sharing – the consumption of an old agent is the same, independent of health status. □

Proof. [Proof of Proposition 14] Part 1 resembles the proof of Proposition 10. The agent's optimal savings rate and health expenditure satisfy the following conditions:

$$\frac{1}{w_t - s_t - g_t} = \delta\sigma_{t+1}\frac{(1+\omega\psi)R_{t+1}}{s_t R_{t+1} + b_{t+1}} + \delta\left(1-\sigma_{t+1}\right)\frac{1}{s_t} \tag{C.12}$$

$$z_{t+1} = \frac{\omega\psi s_t R_{t+1} - b_{t+1}}{1+\omega\psi} \tag{C.13}$$

In the nontrivial steady state with constant tax b, (C.12) implies that the capital level $\overline{k}$ is determined by

$$\frac{1}{(1-\alpha)A\overline{k}^{\alpha} - \overline{k} - \overline{\sigma}b} - \delta\overline{\sigma}\frac{(1+\omega\psi)\alpha A\overline{k}^{\alpha-1}}{\alpha A\overline{k}^{\alpha} + b} - \delta\left(1-\overline{\sigma}\right)\frac{1}{\overline{k}} = 0 \tag{C.14}$$

Rearranging this, we get

$$\frac{1}{(1-\alpha)A\overline{k}^{\alpha} - \overline{k} - \overline{\sigma}b} = \delta\overline{\sigma}\left(1+\omega\psi\right)\left(\frac{1}{\overline{k}} + \frac{\alpha A\overline{k}^{\alpha-1}}{\alpha A\overline{k}^{\alpha}+b} - \frac{1}{\overline{k}}\right) + \delta\left(1-\overline{\sigma}\right), \frac{1}{\overline{k}}$$

which implies that

$$\overline{k} = \frac{(\delta+\delta\overline{\sigma}\omega\psi)\left[(1-\alpha)A\overline{k}^{\alpha} - \overline{\sigma}b\right] + \delta\overline{\sigma}\left(1+\omega\psi\right)\left(\frac{\alpha A\overline{k}^{\alpha}}{\alpha A\overline{k}^{\alpha}+b} - 1\right)\left[(1-\alpha)A\overline{k}^{\alpha} - \overline{k} - \overline{\sigma}b\right]}{1+\delta+\delta\overline{\sigma}\omega\psi}$$

By substituting (C.14), we have

$$\overline{k} = \frac{\delta+\delta\overline{\sigma}\omega\psi}{1+\delta+\delta\overline{\sigma}\omega\psi}\left(1-\alpha\right)A\overline{k}^{\alpha} - \frac{\frac{\delta\overline{\sigma}(1+\omega\psi)b\overline{k}}{(1+\overline{\sigma}\omega\psi)\delta\alpha A\overline{k}^{\alpha}+\delta b(1-\overline{\sigma})} + (\delta+\delta\overline{\sigma}\omega\psi)\overline{\sigma}b}{1+\delta+\delta\overline{\sigma}\omega\psi} \equiv W\left(\overline{k}, b\right) \tag{C.15}$$

(C.15) is equivalent to (3.15) when $b = 0$. Finally, applying the implicit function theorem, the effect of b on the steady-state capital is defined as

$$\frac{\partial\overline{k}}{\partial b} = -\frac{W_b\left(\overline{k}, b\right)}{W_{\overline{k}}\left(\overline{k}, b\right) - 1}$$

It is easy to check that $W_b\left(\overline{k}, b\right) < 0$. Then sign of $\partial\overline{k}/\partial b$ depends on whether $W_{\overline{k}}\left(\overline{k}, b\right)$

is greater or less than 1. Note that $W_{\overline{k}}\left(\overline{k}, b\right)$ corresponds function $M(x)$ in the proof for Proposition 9. Therefore we can follow the same argument that, since the left hand side of (C.15) is 45 degree line, for the initially stable steady state $W_{\overline{k}}\left(\overline{k}, 0\right) < 1$, and for the initially unstable steady state $\overline{k} = 0$, $W_{\overline{k}}\left(\overline{k}, 0\right) > 1$. Hence, the transfer b increases and reduces capital level at initially unstable and stable steady states respectively.

Part 2 follows directly from proof of Proposition 11 with some revision

$$\frac{\partial \overline{U}}{\partial b} = \underbrace{\left(\overline{R}-1\right)\frac{1-\alpha}{(1-\alpha)A\overline{k}^{\alpha}-\overline{k}}\frac{\partial \overline{k}}{\partial b}}_{\text{capital effect}} + \underbrace{\frac{\rho}{\zeta}\frac{\partial \overline{k}}{\partial b}\frac{\partial \overline{U}}{\partial \overline{P}}}_{\text{health effect}}$$

Since $\partial\overline{k}/\partial b < 0$ and $\partial\overline{U}/\partial\overline{P} < 0$, if $\overline{k} \geq \widehat{k}$, the steady-state utility is monotonically increasing in b. □

Proof. [Proof of Proposition 15] We prove this by comparing solutions in optimal pay-as-you-go system to those in the pay-as-you-go system implementing complete risk sharing. At steady state, government's optimization problem is the same as individual agent except it takes account of government budget balance (3.31) and need to choose the policy variable b optimally. Therefore, first order conditions for the government's problem are (C.13), (C.12) and the following one by taking derivative with respect to b and substituting (C.13),

$$\frac{1}{w-s-\sigma b} = \delta\frac{(1+\omega\psi)}{sR+b} \tag{C.16}$$

from which we have

$$b = \frac{\delta(1+\omega\psi)(w-s)-sR}{1+\delta\sigma(1+\omega\psi)} \tag{C.17}$$

Then we can substitute (C.17) into (C.12) to obtain the optimal savings in the optimal pay-as-you-go system

$$s = \frac{\delta(1-\sigma)w}{(1-\sigma R)(1+\delta\sigma\omega\psi)+\delta(1-R\sigma)} \tag{C.18}$$

Now, consider the solutions under the pay-as-you-go system implementing complete risk sharing, which are determined by (C.13) and (C.12). By (C.13), $b = \omega\psi sR$ is necessary for any pay-as-you-go system that ensures complete risk sharing. Then the optimal savings in this system is

$$s = \frac{\delta w}{1+\delta+\delta\sigma\omega\psi R} \tag{C.19}$$

Note (C.18) and (C.19) determine the steady state capital by replacing equilibrium conditions, $s = k$, $w = (1-\alpha)Ak^{\alpha}$ and $R = \alpha Ak^{\alpha-1}$. Moreover, by comparing (C.18) to (C.19), we can find that these two solutions coincides with each other if and only if $R = 1$. Given $R = 1$, we substitute (C.19) and (C.17) into (C.13) and finally have $z = 0$. Note $R = 1$ occurs only when the market solution can restore social optimum. In current stage, without other policy instruments, R can not be equal to 1, which proves the proposition. □

Proof. [Proof of Proposition 16] The sufficiency part relies on definition of constrained demand of assets. Firstly by definition, agent born at $t-1$ being borrowing-constrained at t means $u'\left(c_t^{m,c}\right)/u'\left(c_{t+1}^{o,c}\right) > \beta R_{t+1}$ such that optimal savings at middle-age are binding, i.e. $s_t^c = 0$. In addition, by the condition $\overline{b}_{t-1} = 0$, we have $c_t^{m,c} = \omega^m + f(\omega^y)$ and $c_{t+1}^{o,c} = \omega^o$. Therefore $u'[\omega^m + f(\omega^y)]/u'(\omega^o) > \beta R_{t+1}$.

To prove the necessity part, we know that for all $b_{t-1} \in \left(0, \min\left\{\overline{b}_{t-1}, b_{t-1}^*\right\}\right]$,

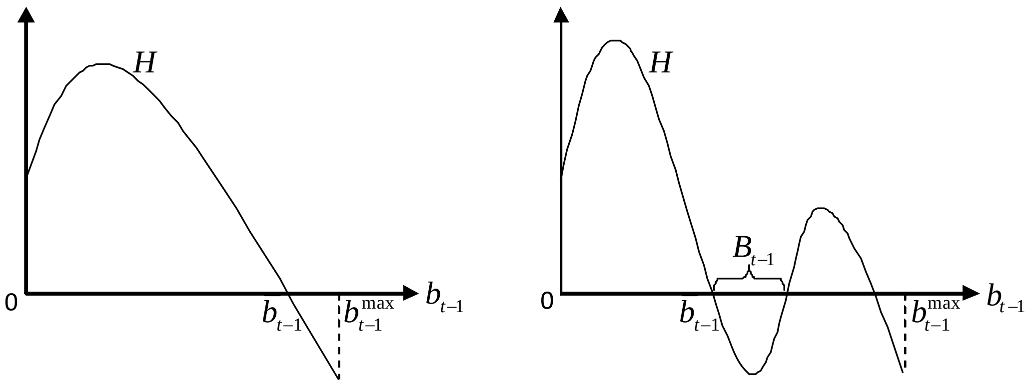
$$\frac{\partial\left[f(\omega^y + b_{t-1}) - R_t b_{t-1}\right]}{\partial b_{t-1}} = f'(\omega^y + b_{t-1}) - R_t \geq 0 \tag{C.20}$$

given any possible borrowing limit $\overline{b}_{t-1} > 0$. Since $u(\cdot)$ is concave and twice continuous, the condition $u'[\omega^m + f(\omega^y)]/u'(\omega^o) > \beta R_{t+1}$ ensures that there must exist some $\widetilde{b}_{t-1} \in \left(0, \min\left\{\overline{b}_{t-1}, b_{t-1}^*\right\}\right]$ such that

$$\frac{u'[\omega^m + f(\omega^y)]}{u'(\omega^o)} \geq \frac{u'\left[\omega^m + f\left(\omega^y + \widetilde{b}_{t-1}\right) - R_t\widetilde{b}_{t-1}\right]}{u'(\omega^o)} > \beta R_{t+1}$$

Therefore for all $b_{t-1} \leq \widetilde{b}_{t-1}$, the optimal savings s_t are binding and equal to zero, which means that middle-aged agent prefers autarky and would default the youthful debt. Given that information, creditor would not set any strictly positive borrowing limit. Hence $\overline{b}_{t-1} = 0$ and, by $u'[\omega^m + f(\omega^y)]/u'(\omega^o) > \beta R_{t+1}$, $s_t^c = 0$. □

Proof. [Proof of Proposition 17] To prove part 1, we need to prove existence of solution $\overline{b}_{t-1}(R_t, R_{t+1})$ and for all $b_{t-1} \in \left[0, \overline{b}_{t-1}\right]$, both IRC1 and IRC2 hold. First we note H is continuously differentiable with $b_{t-1} \in \left[0, b_{t-1}^{\max}\right]$ and it is easy to check that $H\big|_{b_{t-1}=0} > 0$ and $H\Big|_{b_{t-1}=b_{t-1}^{\max}} < 0$. Therefore, H intersects the line $b_{t-1} = 0$ from above at least once and, if there are more than one intersection, as illustrated in Figure C.2, H intersects b_{t-1} axis from above and below alternatively with the first and last one intersected from above. Therefore, there exists either a unique solution $\overline{b}_{t-1}$ as Figure C.2.(a) shows such



(a) Unique solution for H=0

(b) Multiple solutions for H=0

FIGURE C.2. Possible solutions of $\overline{b}_{t-1}$

that for all $b_{t-1} \in \left[0, \overline{b}_{t-1}\right]$, $H \geq 0$, i.e. IRC2 holds, or multiple solutions for $H = 0$ as in Figure C.2.(b). For the case of multiple solutions, $\overline{b}_{t-1}$ is defined by the smallest solution. Otherwise, there always exists a subset $B_{t-1} \subset \left[0, \overline{b}_{t-1}\right]$ such that for any $b_{t-1} \in B_{t-1}$, $H < 0$ and IRC2 is violated.

It remains to verify IRC1, i.e. $s_t^c \geq 0$ for all $b_{t-1} \in \left[0, \overline{b}_{t-1}\right]$, which is equivalent to

$$\frac{u'\left[\omega^m + f\left(\omega^y + b_{t-1}\right) - R_t b_{t-1}\right]}{u'\left(\omega^o\right)} < \beta R_{t+1}$$

Since $u'\left[\omega^m + f\left(\omega^y\right)\right] / u'\left(\omega^o\right) < \beta R_{t+1}$ and $u\left(\cdot\right)$ is concave, it is enough to show, for all $b_{t-1} \in \left[0, \overline{b}_{t-1}\right]$,

$$f\left(\omega^y + b_{t-1}\right) - R_t b_{t-1} > f\left(\omega^y\right) \tag{C.21}$$

Recall that optimal borrowing b_{t-1}^* is defined by $f'\left(\omega^y + b_{t-1}^*\right) = R_t$. Therefore, for all $b_{t-1} \leq b_{t-1}^*$, $f\left(\omega^y + b_{t-1}\right) - R_t b_{t-1}$ is increasing in b_{t-1} and greater than $f\left(\omega^y\right)$. We thus prove (C.21) for all $b_{t-1} \in \left[0, b_{t-1}^*\right]$ and only need to verify (C.21) for any $b_{t-1} \in \left[b_{t-1}^*, \overline{b}_{t-1}\right]$ if $\overline{b}_{t-1} > b_{t-1}^*$. Since $f\left(\omega^y + b_{t-1}\right) - R_t b_{t-1}$ is monotonically decreasing in b_{t-1} for $b_{t-1} \in \left[b_{t-1}^*, \overline{b}_{t-1}\right]$ and $u\left(\cdot\right)$ is concave, it is enough to show

$$\frac{u'\left[\omega^m + f\left(\omega^y + \overline{b}_{t-1}\right) - R_t \overline{b}_{t-1}\right]}{u'\left(\omega^o\right)} < \beta R_{t+1}$$

In the following, we can show it by contradiction.

Suppose $u'\left[\omega^m + f\left(\omega^y + \overline{b}_{t-1}\right) - R_t \overline{b}_{t-1}\right] / u'\left(\omega^o\right) \geq \beta R_{t+1}$. Then given youthful debt

$\overline{b}_{t-1}$, optimal savings of middle-aged agent is $\widetilde{s}_t^* \leq 0$ and we can define V_t^m as

$$V_t^m = u\left[\omega^m + f\left(\omega^y + \overline{b}_{t-1}\right) - R_t\overline{b}_{t-1} - \widetilde{s}_t^*\right] + \beta u\left(\widetilde{s}_t^* R_{t+1} + \omega^o\right)$$

Moreover, by using $u'\left[\omega^m + f\left(\omega^y\right)\right]/u'\left(\omega^o\right) < \beta R_{t+1}$, we can obtain $f\left(\omega^y + \overline{b}_{t-1}\right) - R_t\overline{b}_{t-1} < f\left(\omega^y\right)$ and thus

$$V_t^m < u\left[\omega^m + f\left(\omega^y\right) - \widetilde{s}_t^*\right] + \beta u\left(\widetilde{s}_t^* R_{t+1} + \omega^o\right) \tag{C.22}$$

Since $u'\left[\omega^m + f\left(\omega^y\right)\right]/u'\left(\omega^o\right) < \beta R_{t+1}$, for the middle-aged agent with income $\omega^m + f\left(\omega^y\right)$ and without debt, her optimal savings should be strictly positive. In addition, by second order sufficient condition that $u\left[\omega^m + f\left(\omega^y\right) - s_t\right] + \beta u\left(s_t R_{t+1} + \omega^o\right)$ is concave in s_t, that middle-aged agent's utility value at the point $s_t = 0$ should be greater than or equal to that at the point $\widetilde{s}_t^* \leq 0$

$$u\left[\omega^m + f\left(\omega^y\right) - \widetilde{s}_t^*\right] + \beta u\left(\widetilde{s}_t^* R_{t+1} + \omega^o\right) \leq u\left[\omega^m + f\left(\omega^y\right)\right] + \beta u\left(\omega^o\right) \tag{C.23}$$

By combining (C.22) and (C.23), finally we have

$$V_t^m < u\left[\omega^m + f\left(\omega^y\right)\right] + \beta u\left(\omega^o\right) < u\left[\omega^m + f\left(\omega^y + \overline{b}_{t-1}\right)\right] + \beta u\left(\omega^o\right) \tag{C.24}$$

However, according to the definition of $\overline{b}_{t-1}$, $V_t^m = u\left[\omega^m + f\left(\omega^y + \overline{b}_{t-1}\right)\right] + \beta u\left(\omega^o\right)$ which contradicts with (C.24).
Part 2 follows directly from Figure C.2. □

Proof. [Proof of Corollary 18]

Applying implicit function theorem and envelop theorem in equation (4.17), it is straightforward to have

$$\begin{aligned}
\frac{\partial \overline{b}_{t-1}}{\partial R_t} &= -\frac{-\overline{b}_{t-1}}{\left.\frac{\partial H}{\partial b_{t-1}}\right|_{b_{t-1}=\overline{b}_{t-1}}} < 0 \\
\frac{\partial \overline{b}_{t-1}}{\partial R_{t+1}} &= -\frac{s_t^c}{\left.\frac{\partial H}{\partial b_{t-1}}\right|_{b_{t-1}=\overline{b}_{t-1}}} > 0
\end{aligned}$$

Similarly, we can obtain

$$\frac{\partial \bar{b}_{t-1}}{\partial \omega^y} = -\frac{f'\left(\bar{b}_{t-1}+\omega^y\right)\left[u'\left(c_t^{m,c}\right)-u'\left(c_t^{m,d}\right)\right]}{\frac{\partial H}{\partial b_{t-1}}\Big|_{b_{t-1}=\bar{b}_{t-1}}} > 0 \quad (C.25)$$

$$\frac{\partial \bar{b}_{t-1}}{\partial \omega^m} = -\frac{\left[u'\left(c_t^{m,c}\right)-u'\left(c_t^{m,d}\right)\right]}{\frac{\partial H}{\partial b_{t-1}}\Big|_{b_{t-1}=\bar{b}_{t-1}}} > 0 \quad (C.26)$$

$$\frac{\partial \bar{b}_{t-1}}{\partial \omega^o} = -\frac{\beta u'\left(s_t^c R_{t+1}+\omega^o\right)-\beta u'\left(\omega^o\right)}{\frac{\partial H}{\partial b_{t-1}}\Big|_{b_{t-1}=\bar{b}_{t-1}}} < 0 \quad (C.27)$$

where $c_t^{m,c} = \omega^m + f\left(b_{t-1}+\omega^y\right) - R_t b_{t-1} - s_t^c$ and $c_t^{m,d} = \omega^m + f\left(b_{t-1}+\omega^y\right)$. Since $c_t^{m,c} < c_t^{m,d}$ and $s_t^c \geq 0$, the signs of $\partial \bar{b}_{t-1}/\partial \omega^i$, $i = \{y, m, o\}$, are straightforward. □

Proof. [Proof of Proposition 19] It is evident that Part 2 follows directly from Part 1. Therefore we only need to prove Part 1. Define total resource available for young agent x by a function $F\left(\tau^y\right)$, i.e. $x = F\left(\tau^y\right) = \bar{b}\left(\tau^y\right) + \tau^y$. Then to prove necessity part, we need to prove that for all $\omega^m \in [\widehat{\omega}^m, \overline{\omega}^m)$, there exists τ^y such that both resource condition, i.e. $F\left(\tau^y\right) \geq x^*$, and consumption smoothing condition, i.e. $\tau^y \leq \widehat{\tau}^y$, are satisfied. Note, by definition $F\left(\tau^y\right) \geq \tau^y$ always holds. Therefore if we can prove $\widehat{\tau}^y\left(\omega^m\right) > x^*$ for all $\omega^m \in [\widehat{\omega}^m, \overline{\omega}^m)$, we would have $F\left(\tau^y\right)|_{\tau^y=\widehat{\tau}^y} = \widehat{\tau}^y \geq x^*$ for all $\omega^m \in [\widehat{\omega}^m, \overline{\omega}^m)$, and then by using $F\left(\tau^y\right)|_{\tau^y=\widehat{\tau}^y} \geq x^* > F\left(\tau^y\right)|_{\tau^y=0}$ and intermediate value theorem, we can conclude that for any $\omega^m \in [\widehat{\omega}^m, \overline{\omega}^m)$ there must exist $\tau^y \in (0, \widehat{\tau}^y]$ such that $F\left(\tau^y\right) = x^*$, which proves the necessity part. Now we prove $\widehat{\tau}^y\left(\omega^m\right) > x^*$ for all $\omega^m \in [\widehat{\omega}^m, \overline{\omega}^m)$. Firstly by applying implicit function theorem in (4.25), we can obtain

$$\frac{\partial \widehat{\tau}^y}{\partial \omega^m} = -\frac{u''\left(\omega^{m\prime}\right)u'\left(\omega^{o\prime}\right)}{u''\left(\omega^{m\prime}\right)u'\left(\omega^{o\prime}\right)\left[f'\left(\widehat{\tau}^y\right)-1-R\right]-Ru''\left(\omega^{o\prime}\right)}$$

Evidently if $f'\left(\widehat{\tau}^y\right) \leq (1+R)$, $\partial \widehat{\tau}^y/\partial \omega^m > 0$. Secondly consider $\omega^m = \widehat{\omega}^m$, where by definition $f'\left(\widehat{\tau}^y\right) = f'\left(x^*\right) = (1+R)$, we must have

$$\frac{\partial \widehat{\tau}^y}{\partial \omega^m}|_{\omega^m=\widehat{\omega}^m} > 0$$

Third, according to (4.25) and (4.28), the solution of ω^m to ensure $\widehat{\tau}^y\left(\omega^m\right) = x^*$ is unique and equal to $\widehat{\omega}^m$. Then we can conclude that if and only if $\omega^m \in (\widehat{\omega}^m, \overline{\omega}^m)$, $\widehat{\tau}^y\left(\omega^m\right) \geq x^*$. Otherwise, there would exist more than one solution for $\widehat{\tau}^y\left(\omega^m\right) = x^*$ which is impossible

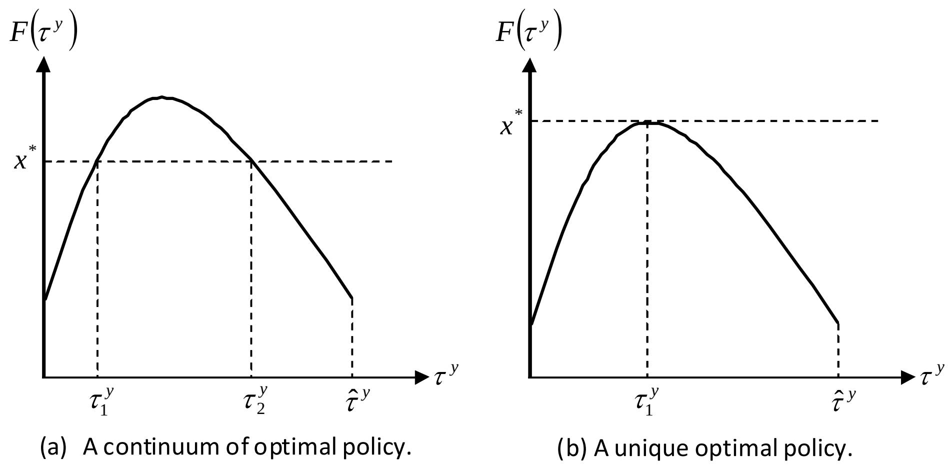


FIGURE C.3. Two cases for the subset $[\tau_1^y, \tau_2^y]$

as discussed.

The sufficiency part is equivalent to the statement that optimal $\tau^y \geq 0$ does not exist if $\omega^m \in (\omega^o, \widehat{\omega}^m)$, where ω^o is by assumption the low bound of ω^m. I prove it by contradiction. Firstly note $\widehat{\tau}^y(\omega^m) < x^*$ for any $\omega^m \in (\omega^o, \widehat{\omega}^m)$. Therefore we have $F(\tau^y)|_{\tau^y = \widehat{\tau}^y} = \widehat{\tau}^y < x^*$ for any $\omega^m \in (\omega^o, \widehat{\omega}^m)$. Now suppose optimal $\tau^y \geq 0$ exists for some $\widetilde{\omega}^m \in (\omega^o, \widehat{\omega}^m)$. Recall existence of optimal $\tau^y \geq 0$ requires resource condition $F(\tau^y) \geq x^*$ and consumption smoothing condition $\tau^y < \widehat{\tau}^y$. Since $F(\tau^y)|_{\tau^y = \widehat{\tau}^y} < x^*$ holds for any $\omega^m \in (\omega^o, \widehat{\omega}^m)$, for $\widetilde{\omega}^m$ there must exist a subset $[\tau_1^y, \tau_2^y] \subset (0, \widehat{\tau}^y)$ such that for all $\tau^y \in [\tau_1^y, \tau_2^y]$, complete market solution can be restored, i.e. $F(\tau^y) \geq x^*$, and $F(\tau^y)\big|_{\tau^y = \tau_1^y} = F(\tau^y)\big|_{\tau^y = \tau_2^y} = x^*$. The set $[\tau_1^y, \tau_2^y]$ has two possible cases, a continuum of optimal policy $\tau_1^y < \tau_2^y$ and a unique optimal policy $\tau_1^y = \tau_2^y$ as shown in Fig. C.3. Their corresponding proofs are different.

If $\tau_1^y < \tau_2^y$, then we must have $F'(\tau^y)\big|_{\tau^y = \tau_1^y} > 0$ and $F'(\tau^y)\big|_{\tau^y = \tau_2^y} < 0$. Moreover, since τ_1^y and τ_2^y replicate complete market solution, for $\tau^y = \{\tau_1^y, \tau_2^y\}$, $u'(c^{m,c}) = \beta R u'(c^{o,c})$ and $f'(\bar{b} + \tau^y) = f'(x^*) = R$. Substituting these two equations in (4.24), we have, for $\tau^y = \{\tau_1^y, \tau_2^y\}$,

$$F'(\tau^y) = 1 + \frac{\partial \bar{b}}{\partial \tau^y} = 1 + \frac{1}{R} - \beta \frac{u'(\omega^o + R\tau^y)}{u'[\omega^m - (1+R)\tau^y + f(x^*)]}$$

and $F''(\tau^y) > 0$. Therefore, if $F'(\tau^y)\big|_{\tau^y = \tau_1^y} > 0$, then we must have $F'(\tau^y)\big|_{\tau^y = \tau_2^y} > 0$ which contradicts with the fact $F'(\tau^y)\big|_{\tau^y = \tau_2^y} < 0$. If $\tau_1^y = \tau_2^y = x^*$, then $F(\tau^y)\big|_{\tau^y = \tau_1^y}$

must be a local maximum, i.e. $F'(\tau^y)\big|_{\tau^y=\tau_1^y}=0$ and $F''(\tau^y)\big|_{\tau^y=\tau_1^y}\leq 0$. Some algebra leads to

$$F''(\tau^y)\big|_{\tau^y=\tau_1^y}=\frac{\begin{array}{c}\left\{\left[\beta R^2u''(c^o)+u''(c^m)\right]\left(1+\frac{\partial s^*}{\partial\tau^y}\right)-\beta R^2u''\left(c^{o,d}\right)\right\}f'(x^*)\,u'\left(c^{m,d}\right)\\ -\beta R(1+R)\,f'(x^*)\,u'\left(c^{o,d}\right)u''\left(c^{m,d}\right)\end{array}}{\left[f'(x^*)\,u'\left(c^{m,d}\right)\right]^2}$$

Since $u'(\omega^o+R\tau^y+Rs^*)/u'\left[\omega^m+f(x^*)-(1+R)\tau^y-R\bar{b}-s^*\right]=\beta R$, we have

$$\frac{\partial s^*}{\partial\tau^y}\big|_{\tau^y=\tau_1^y}=-1$$

By substituting the above result into $F''(\tau^y)\big|_{\tau^y=\tau_1^y}$, we finally get

$$F''(\tau^y)\big|_{\tau^y=\tau_1^y}=\frac{-\beta Rf'(x^*)\left[R^2u''\left(c^{o,d}\right)u'\left(c^{m,d}\right)+(1+R)\,u'\left(c^{o,d}\right)u''\left(c^{m,d}\right)\right]}{\left[f'(x^*)\,u'\left(c^{m,g,d}\right)\right]^2}>0$$

which contradicts with the fact $F''(\tau^y)\big|_{\tau^y=\tau_1^y}\leq 0$. □

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